ARTIS ANALYTICAE PRAXIS,

Ad æquationes Algebraïcas nouâ, expeditâ, & generali methodo, resoluendas:

TRACTATVS

E posthumis THOMÆ HARRIOTI Philosophi ac Mathematici celeberrimi schediasmatis summâ fide & diligentiâ descriptus:

ET

ILLVSTRISSIMO DOMINO
DOM. HENRICO PERCIO,
NORTHVMBRIÆ COMITI,

Qui hæc primò, sub Patronatus & Munificentiæ suæ auspicijs ad proprios vsus elucubrata, in communem Mathematicorum vtilitatem, denuò reuisenda, describenda, & publicanda mandauit, meritissimi Honoris ergò Nuncupatus.

LONDINI
Apud ROBERTVM BARKER, Typographum Regium: Et Hæred. IO. BILLII.
Anno 1631.

MVSEVM BRITANNICVM

XI
e
4 h

PRÆFATIO AD ANALYSTAS.

ARTIS ANALYTICÆ, cuius causa hîc agitur, post eruditum illud Græcorum sæculum antiquatæ iamdiù & incultæ iacentis, restitutionem *Franciscus Vieta*, Gallus, vir clarissimus, & ob insignem in scientijs Mathematicis peritiam, Gallicæ gentis decus, primus singulari consilio & intentato antehàc conamine aggressus est; atque ingenuam hanc animi sui intentionem per varios tractatus, quos in argumenti huius elaboratione eleganter & acutè conscripsit, posteris testatam reliquit. Dùm verò ille veteris Analytices restitutionem, quam sibi proposuit, seriò molitus est, non tàm eam restitutam, quàm proprijs inuentionibus auctam & exornatam, tanquàm nouam & suam, nobis tradidisse videtur. Quod generali conceptu enuntiatum paulò fusiùs explicandum est; vt, ostenso eo quod primùm à *Vieta* in instituto suo promouendo actum est, quid posteà ab authore nostro doctissimo *Thomâ Harrioto*, qui illum certamine isto Analytico sequutus est, præstitum sit, meliùs innotescere possit.

Quarè vt rem ab initio repetamus; Veteres illos Artifices, in Problematum solutionibus inuestigandis, quorum deductiones ordinis Quadratici limites non excedunt, Analyticen communiter exercuisse, in varijs ipsorum monumentis tùm effectu manifestum, tùm disertè ab ipsis significatum est. Vnde scientias Mathematicas, quas ab illis accepimus, artis huius inuentricis beneficio, quamplurimis accessionibus locupletatas fuisse, pro certo existimandum est. Nam Problemate processu Analytico ad solutionis statum deducto, liberum & facile eis fuit, facto per Analysis vestigia regressu, demonstrationem syntheticè construere, constructamque, suppressâ Analysi, Problemati attexere. Sed priuilegium hoc eis intra communium Elementorum terminos, siuè (vt ipsi loquuntur) in loco plano versantibus, concessum tantummodo fuit. Cùm autem tentatâ Analysi in sublimiorum ordinum formulas (vt cubicas præsertim) incidere eis obtigit; votis suis minùs prosperè succedente solutione, nè omni artis subsidio ad eam formâ aliquâ Geometricâ prestandam destituti viderentur; vel ad locos solidos (quo nomine sectiones Conicas intellexere) vel ad locos quos lineares vocarunt, (vt sunt

Helice

Helice, Conchoides, Quadrantica, & huius generis similia) tanquàm ad postulata artis defectuosæ supplementa, confugere solebant. Sunt autem supplementa ista delineationes quædam tortuosæ per motus compositos mechanicè descriptæ, calculi aut ratiocinij vlterioris, quàm quod ex præsupposita ipsarum genesi immediatè dependet, omninò incapaces. Vnde factum est, vt earum adhibito adminiculo, desiderata problematis solutio manus & oculi officio organicè tantùm expedienda erat. Atq; in huiusmodi statu hæsit veterum Græcorum in problematis soluendis facultas Analytica, quamdiù artium Mathematicarum studium & professio apud eos floruit.

Deuictâ verò tandem Barbarorum armis Græciâ, & in seruitutem redactâ, vniuersa Græcorum literatura ad Arabum scholas transmigrauit. Vbi, per succedentis sæculi tempora, gentis ingeniosæ studijs summoperè exculta & amplificata est. Quanquam autem in alijs philosophiæ partibus multa quidem vtilia, atq; nonnulla etiam abstrusiora solerti eorum indagine inuenta, ad nos peruenerint; ac tametsi Arabicum ipsius *Algebræ* nomen ab eis impositum, (præter scripta eorum paucula in eo genere extantia) artis apud eos studium & praxim viguisse argumentum euidens sit; vnus tamen *Diophantus* Analysta Græcus, ex antiqua Mathematicorum familia superstes, obstat, quò minùs vel *Algebræ* inuentionem, vel quicquam, quod ad Analyticen perficiendam vel augendam faciat, Græcorum inuentis superadditum, Arabibus acceptum referre teneamur.

Pristina igitur Græcorum Analytice, eodem prorsus quo ab ipsis relicta est imperfectionis statu, per Arabum manus intacta ad nostra vsque tempora deuoluta permansit. Dum *Cardanus* & *Tartaglia*, Itali, celebres superioris ętatis Mathematici, & *Algebræ* studiosissimi, fundamēto quodā Geometrico innixi, (de inuētionis gloria magnoperè inter se discertantes) artē, ad Cubici ordinis æquationes apodicticè resoluendas promouere conati sunt; casus nonnullos conditionatos accuratè quidem, sed formâ Binomijs radicalibus admodum perplexâ, resoluendo. De conditionatis dictum est, quia resolutionis fundamentum hoc generale & absolutum non est. Post hos alij inuentum istud eorum ad incudem reuocarunt; inter quos *Steuinus* Belga in Arithmetica sua generali omnium optimè & diligentissimè materiam hanc pertractauit. Primò, æquationum Cubici generis, quæ naturâ & conditione suâ primariâ resolubiles sunt, (quarum scilicet resolutio ex supposito fundamento immediatè extrui potest) resolutionis modum proponit. Secundò, æquationum Cubicarum formas illas quæ conditione suâ ad primarias reducibiles sunt, reducit & resoluit. Tertiò, Biquadraticas quoque, ad primarias cubicas reducibiles reductas, itidem resoluit; reliquis, tam Cubici quàm Biquadratici generis non conditionatis, (quæ totius multitudinis pars magna est) pro irresolubilibus, maximo artis detrimento, tacitâ exclusione damnatis. Atq; hic Italici huius inuenti progressus & terminus fuit, non tàm ignorantiâ nostrâ quàm ipsâ rei naturâ præfinitus.

Prodijt

Prodijt autem tandem *Vieta*, magnus ille in Analyticis architectus, qui quùm, varijs adhibitis Supplementorum, Recognitionum, atque Angularium sectionum subsidijs, omnia tentasset quibus, tanquàm ingenij sui machinis, inuictam hanc artis Analyticæ anomaliam superaret; haud longè tamen vltra præfatum antecessorum suorum terminum rem prouexisse videtur; donec frustrà tentatis Geometricis, in Arithmetico genere insistens, *Exegeticen* suam numerosam feliciter excogitauit. Quâ demùm inuentâ, fastuosum illud & vniuersale problema suum, *Nullum non problema soluere*, fidenter asseuerare potuit. Est enim Ars illa, ad omnes omnium ordinum & formarum æquationes generali, vniformi ac infallibili methodo resoluendas, ab ipsa natura ordinata. Quùm verò problematum solutiones æquationum resolutionibus finaliter perficiantur; *Vieta* idcirco, immensâ *Exegetices* huius in æquationibus resoluendis potestate perspectâ, vniuersalem problematum solutionem illius ministerio possibilem existimans, magnificâ huiusmodi enunciationis formâ Problema illud insignire voluit. *Exegetices* huius inuentum, eorum quæ à *Vieta*, ad opus restitutorium ab ipso conceptum, collata sunt, dignitate præcipuum, narrationis ordine primum esto.

Restat alterum ipsius inuentum in scholam Mathematicam, titulo *Logistices* Speciosæ introductum: quæ licèt Analytices restitutionem minùs essentialiter quàm *Exegetice* numerosâ, attingat; tamen, cùm naturæ prioritate, ac proindè vsus generalitate, illam longè superet, non minoris æstimanda est. Veteres sane *Logistice* hâc speciosâ non sine maximo dispendio caruisse, agnouerit quisquis incredibilem illius commoditatem in materia Mathematica compendiosè & dilucidè tractanda, præ verbosa veteris stili grauitate, expertus fuerit. Quoniam igitur duobus hisce auctarijs, Logistice Speciosâ atque *Exegetice* numerosâ, (quarum in veterum scriptis nè vestigium quidem vllum extat) *Vietam* artem locupletasse constet; nouam eam potiùs, vt dictum est, magna saltem ex parte fecisse, quàm veterem restituisse, non immeritò censendus est.

Exegetice ista numerosa est quam hîc proferimus, è *Thomæ Harrioti* nostri schediasmatis depromptam; non quidem vt primis *Vietæ* cogitationibus formata est, sed posterioribus *Harrioti* ita reformatam, vt si *Vieta Exegetices* inuentione Analyticen nouam quodammodo fecisse visus fuerit, *Harriotum Exegetices* recognitione ipsum *Vietam* nouum, nouo certè ac multò magis expedito & ad vsum facto habitu conspiciendum produxisse facilè iudicauerint, qui vtriusque institutionis formas ad praxim reuocatas comparauerint.

Ad *Exegetices* autem reformationem istam perficiendam, Logistices quoque *Vietæam* formam priùs mutatam esse omninò necessarium ei fuit. Quam enim *Vieta* notis interpretatis exercendam præcepto & exemplo proposuit,

licèt

licèt ad nouæ disciplinæ intelligentiam vtilis esse potuit, ad ordinariam tamen praxim incommoda posteà reperta est.

Harriotus igitur solâ literali notatione, Elementis scilicet vel simplicibus vel vtcunque combinatis, pro calculi aut ratiocinij exigentia vsus est. Opportunâ quidem hâc mutatione, *Logistices* speciosæ aliquatenùs molestam anteà & minùs concinnam praxim ad summam tùm facilitatem tùm perspicuitatem redactam esse, multiplicibus præsentis tractationis exemplis patefactum est. Logistices certè huius dexteritate fretus *Harriotus*, *Exegetices* reformationem duobus precipuè inuentis suis molitus est. Primò, æquationes quasdam è radicibus Binomijs generatas constituit, quas Canonicas appellat. Harum ad æquationes communes factâ applicatione, siqua radicum ambiguitas communibus subsit, per Canonicarum istarum æquipollentiam inuento admodùm ingenioso detegitur ac determinatur. Secundò, quùm ad ipsam *Exegetices* numerosæ praxim deuentum est, species quasdam polynomias ex ipsis æquationum resolutioni propositarum speciebus deducit; quas item Canonicas vocat. Sunt enim reuerà ipsius resolutionis Canones siuè regulatrices quarum vniformiter continuatâ applicatione, operis Analytici processus à principio ad finem tantâ facilitate ac certitudine dirigitur, vt *Harriotus* vnicâ huius artificij, præ cæteris huius generis inuentis suis, inuentione, *Exegeticen* numerosam (artem Mathematicarum omnium instrumentariam, atque eo nomine vtilissimam) ad absolutam perfectionem redegisse verè existimandus sit. Atque hæc ferè sunt quæ ab Authore nostro in *Exegetices* reformatione elucubrandâ peracta sunt, summatim quidem dicta, sed quæ in sequenti tractatu maximo Analystarum commodo particulatim & diligenter explicata sunt.

DEFINITIONES.

Definitiones quædam, quæ proœmij loco vocabulis tam ipsius artis communibus, quam præsentis tractatus peculiaribus interpretandis vtiliter inseruire possint.

Definitio Prima.

L*Ogistice speciosa*, cuius in Analyticis istis frequens, & omnino necessarius est vsus Arithmeticæ, generis eiusdem partitipatione germana est. Est namque Arithmetica Logistice numerosa. Hoc tantum inter eas (quantum ad nominis rationem attinet) discriminis est. In Arithmetica rerum mensurabilium quantitates per characteres seu figuras artis proprias, numero, vt generali mensurâ exprimuntur & computantur. In hac autem per notas literales, elementa scilicet alphabetica, res ipsæ tanquam in specie (ex vsu forensi recepto speciei vocabulo) significantur & omnimodè tractantur, vnde *Speciosa* appellationem obtinuit.

Definitio 2.

Aequatio communi significatione pro quantitatum duarum vel plurium qualicunque æqualitate vsurpatur, sed vt est proprium huius artis vocabulum est quantitatis quæsitæ cum quantitate aliqua data, factâ alterius ad alteram comparatione, distinctè ordinata æqualitas. Cuius pars quæsititia est potestas pura vel affecta, pars vero data homogeneum comparationis seu æquationis datum nominari solet.

Definitio 3.

In propositionibus cuiuscunque generis iisque siue theorematis siue problematis scientificè cõstituendis, potissima demonstrandi methodus, & via omnino naturalis est, quâ à principijs & elementis doctrinæ cuiusq; proprijs per continuatas consequentias componendo proceditur ad propositi confirmationem, vnde methodus compositiua & veterum artificum idiomate Synthetice appellata est.

Definitio 4.

Quoniam autem sæpissime accidit in problematis præsertim fortuitò oblatis soluendis, vt Logista medijs ad propositum arguendum idoneis destitutus, à scientiæ principijs & elementis naturali synthetices viâ ad problematis solutionem ratiocinio exquirendam & cõcludendam procedere nequeat. In huiusmodi ignorationis casu qui fere perpetuus est, necessitate edoctus viam capessit retrogradam & naturali contrariam. Facto namque initio ab ignota aliqua & quæsita ad problema pertinente quantitate, tanquam notâ & datâ assumptâ, continuatis consequẽtijs resoluendo progreditur quousq; in assumptę illius quantitatis tanquam datæ (siue simplicis siue graduatæ & affectionibus implicatæ) cum quantitate aliqua veré data æqualitatem incidat. Ex qua quidem æqualitate artificio huiusmodi inuenta & rite constitua ipsa de qua quæritur quantitas, vel per se manifesta prodeat, vel vlteriore artificio eruatur vnde problema tandem soluatur. Atque methodus ista resolutiua est quam vocabulo significante veteres Analyticen appellarunt.

DEFINITIO 5.

Componendi & resoluendi voces quæ duabus hisce definitionibus inseruntur Mathematicorum solennes sunt, quibus in demonstrationibus suis conficiendis contrarias ratiocinandi vias, vbi opus fuerit, expressè significare solent. In earum altera à simplicioribus & minùs compositis ad magis composita, quod est componendo, secundùm naturalem scientiarum structuram & ordinationem, à priore scilicet ad posterius descenditur. In altera verò à magis compositis ad minùs composita & simplicia, quod est resoluendo, ordine retrogrado & naturali contrario, à posteriore scilicet ad prius ascenditur ad conclusionem. Nam scientiarum elementa & axiomata si legitimè constituta sint, conuertibilia esse debent; vnde fit, vt quæ naturâ antecedentia sunt, ratione consequentia esse possint. Atque idcirco processum Logicum per consequentia in vtramque partem æquè firmum & apodicticum esse necesse est.

DEFINITIO 6.

Ex superiore Analytices definitione duas illius officio distinctas partes esse colligitur. Quarum prima in æquationum constitutione tota versatur, quæ scilicet vt dictum est, ab assumpto quæsito tantum dato per consequentia tendit ad assumpti quæsiti cum quantitate aliqua data æquationem inueniendam & constituendam, & in constituta terminatur & acquiescit. Cum autem æquationum huiusmodi constitutio in artificiosa quæsiti inuestigatione consistat, veteres artem istam, Zeteticen, quasi inuestigatoriam seu inquisitoriam nominarunt.

DEFINITIO 7.

Analytices pars altera est, quâ ex æquatione per Zeteticen iam constituta, quæsita quantitas continuato vel mutato resolutionis genere exhibetur, vel specie scilicet si reipsa exhiberi possit, vel numero, si numero explicanda sit, vnde propositi problematis perfecta tandem existit solutio. Huic Analytices parti à *Francisco Vieta*, magno artis Analyticæ magistro, *Exegetices*, quasi declaratoriæ seu exhibitoriæ nomen impositum est.

DEFINITIO 8.

Veteres Analystæ præter Zeteticen quæ ad problematum solutionem propriè pertinet aliam Analycices speciem fecerunt poristicen, quasi illatoriam quâ theorematum fortuito propositorum dubitata veritas examinatur. Methodus enim vtriusque Analytica est, ab assumpto probando tanquam concesso per consequentia ad verum concessum. In hoc tamen inter se differunt quod Zetetice quæstionem deducit ad æquale, datum scil. quæsito, poristice autem ad idem, vel concessum, vt in exemplis vtriusque generis videre licet. Vnde & altera inter eas oritur differentia quòd in poristice, cum processus eius terminetur in identitate vel concesso, vlteriore resolutione non sit opus (vt fit in Zetetice) ad propositi finalem verificationem.

DEFINITIO 9.

Ex præmissa *Exegetices* definitione, duplicem eam esse debere apparet iuxta duplicem Logistices naturam, numerosam & speciosam, quæ licet eandem tractent materiam, & ad eundem finem, æquationum scilicet iam constitutarum resolutionem spectent, in praxi tamen & operationis modo toto genere inter se differunt. Nam Exegetice speciosa æquationem Zetetices operâ primo constitutam, continuato ratiocinationis processu ad speciem siue formam resolutioni propriè ordinatam reducit, & ex ordinata quæsitam quantitatem in specie sua reali certo ac simplici artificio exhibet. Adeo vt Exegetice ista dum in æquationibus exerceturquæ ordinem quadraticum non excedunt, sed in loco plano, vt loquuntur veteres, subsistunt, propter exactam methodi vniformitatem ac

certitudinem

certitudinem pro perfectè scientifica habenda sit. Cum autem tentatum est à modernis quibusdam Algebræ authoribus deficiente priore istâ methodo alio inuento fundamento ab sublimiorum graduum æquationes, cubicas scilicet & biquadraticas resoluendas, artem promouere, mutilam eam & imperfectam ex inemendabili fundamenti sui imperfectione nobis reliquerunt, vt æquationum quæ in altiores illas formulas incidunt, pars magna pro irresolubilibus habitæ sint.

Definitio 10.

Excogitata est igitur tandem altera illa Exegetice numerosa, quæ ad omnes omnium ordinum æquationes resoluendas extenditur, methodo generali ac infallibili, quâ scilicet ex æquatione quacunque resolutorio Zetetices processu constitutâ & ad numeros reuocatâ, quæsita quantitas, secundariâ mutatæ resolutionis operâ numero exhibetur. Peculiaris est Exegetices huius ars, regulis suis & præceptis ad praxim instructa, quæ in præsenti tractatu, qui totus Exegeticus est, traduntur.

Definitio 11.

Resolutionem Exegetices numerosam ad præuiam zetetices resolutionem comparatam, secundariam appellare visum est, vt ex adiecta communi nomini differentia vtramque licet diuersi generis, analysim tamen siue resolutionem esse constet. Zeteticen quidem Logicam siue discursiuam, Exegeticen vero operatiuam. Non est enim aliud exegetice ista numerosa quam veteris Arithmeticæ operationis, quæ in simplicium tantùm potestatum radicibus extrahendis communiter vsitata hucusque permansit, noua quædam & à veteribus intentata ad methodi generalitatem exaltatio.

Definitio 12.

Radicis vocabulum duplici significatione in sequentibus vsurpatur. Nam in æquationibus zeteticè constituendis ipsum perpetuò est quæsitum quod in ratiocinationis initio pro concesso assumitur seu supponitur, vnde in iam constitutis dum adhuc in æquationis inuolucris ignota latet vel potestatis vel suo modo significanter, etiam æquationis radix quæsititia vel supposititia nominari potest. In æquationibus autem resoluendis siue iam resolutis, cùm per exegeticen, seu speciosam ex ipsa æquatione iam constituta ratiocinio analytico in specie sua reali exhibita est, seu numerosam è dato æquationis homogeneo operatione analyticâ in numero expresso educta est, radicem exhibititiam, vel eductitiam eam pro vario resolutionis genere, variato nomine appellare liceat, quæ etiam radicis quæsititiæ valor communiter nominari obtinuit. Ac præterea quia de ea iam exhibita vel educta æquatio ad operis vel ratiocinij verificationem explicari solet, radix æquationis explicatoria dici quoque potest. Non est enim potestas è qua resolutionis via exhibita vel educta est radix, vel quæ de radice iam exhibita vel educta via compositionis explicatur, sed ipsa æquatio. Quare licèt in *Vietæ* exegeticis vbi disertè de quadratis & cubis & reliquis potestatibus problemata enunciata sunt, de potestatis radice necessario quærendum erat: Tamen in sequentibus quæ non de potestatibus sed de ipsis æquationibus subiectiuè & integrâ specie acceptis enunciantur, in quibus non magis summæ potestatis quàm inferiorum graduum radix est, de radice æquationis explicatoria, vel saltem de radicis quæsititiæ valore quæstionem institui, enunciatis consonantius esse videtur. Cum autem radix æquationis explicatoria eadem quoque potestatis generatoria sit, siue hoc siue illo nomine inscribatur rem ipsam intelligentibus nihil interesse posse de quo curandum sit, manifestum est.

Definitio 13.

Præmissa æquationis definitio de æquatione per zeteticen rite constituta accipienda est, quę quoniam à problematum seu quęstionum vtcunque propositarum terminis communiter

muniter deducitur æquationem communem siue aduentitiam nuncupare licet, vt ipso nomine ab alio æquationis genere prorsus diuerso distinguatur, quam canonicam appellare visum est, de qua postea.

Definitio 14.

Est etiam aliud quoddam æquationum genus; quæ licet canonicæ non sint, cum tamen ab ijs, vt ab originalibus suis æquationes canonicæ deriuentur, canonicarum originales in sequentibus denominantur. Huius generis æquationes per genesim siue multiplicationem e radicibus binomiis absque alio discursu immediatè conficiuntur. In quibus facta ex radicibus multiplicatis, radicibus ipsis multiplicandis sub forma tantùm multiplcationis ordinatis æquantur, vt in exemplis hisce videre est

$$\begin{array}{l|l} a+b & = aa+ba \\ a-c & \qquad -ca-bc \end{array}$$

$$\begin{array}{l|l} a+b & = aaa+baa+bca \\ a+c & \qquad +caa-bda \\ a-d & \qquad -daa-cda-bcd \end{array}$$

$$\begin{array}{l|l} a+b & = aaaa+baaa+bcaa \\ a+c & \qquad +caaa+bdaa \\ a+d & \qquad +daaa+cdaa+bcda \\ a-f & \qquad -faaa-bfaa-bcfa \\ & \qquad\qquad -cfaa-bdfa \\ & \qquad\qquad -dfaa-cdfa-bcdf \end{array}$$

Harum æquationum forma propriæ æquationis definitioni minimè quadrat. Quæ tamen ab ijs deriuantur duæ canonicarum species, primaria scilicet & secundaria, tum definitioni conformes sunt, tum vsui propriè applicabiles.

Definitio 15.

Primaria canonicarum species est earum quæ ab originalibus per deriuationem constituuntur. Nam reiectâ formali radicum ordinatione, quæ æquationis originalis pars altera est, & ex alterius partis homogeneis homogeneo dato per affectionis mutationem in situm reliquis oppositum translato, fit æquatio canonica primaria. Deriuationis ratio in sectione 2, proponitur, speciei autem exempla hęc sunt.

$$\begin{array}{l} aa+ba \\ \quad -ca = +bc \end{array}$$

$$\begin{array}{l} aaa+baa+bca \\ \quad +caa-bda \\ \quad -daa-cda = +bcd \end{array}$$

$$\begin{array}{l} aaaa+baaa+bcaa \\ \quad +caaa+bdaa \\ \quad +daaa+cdaa+bcda \\ \quad +faaa-bfaa-bcfa \\ \qquad\quad -cfaa-bdfa \\ \qquad\quad -dfaa-cdfa = +bcdf \end{array}$$

Definitio 16.

Secundaria canonicarum species est earum quæ à primarijs per reductionem constituuntur. Nam tollendo vnum aliquem ex primariæ gradibus parodicis fit secundaria seu reductitia varij sunt huius speciei reductionis modi, de quibus in tertia sectione agitur, speciei autem exempla huiusmodi sunt.

$$aa = + bb$$

$$\begin{array}{l} aaa - bba \\ \quad\; - bca \\ \quad\; - cca = + bbc \\ \qquad\qquad\quad + bcc \end{array}$$

$$\begin{array}{llll} aaaa & - bbaa & - bbca & \\ & - bcaa & - bcca & \\ & - ccaa & - bbda & \\ & - bdaa & - bdda & \\ & - cdaa & - ccda & \\ & - ddaa & - cdda & \\ & & - 2.bcda = & + bbcd \\ & & & + bccd \\ & & & + ccdd \end{array}$$

Definitio 17.

Hæ duæ æquationum species pro canonicis habentur, quia per earum applicationem tanquam per canones siue regulas, radicum numerus in æquationibus communibus determinatur, (quod in quarta Sectione videre licet) vnde non à constitutionis forma sed ab instrumentario huiusmodi vsu canonicarum nomen illis debetur.

Definitio 18.

AEquatio reciproca appellatur, cuius homogeneum datum facto è coefficientibus: & reciprocè potestas facto è gradibus parodicis æquatur. Qualis est $aaa - caa + bba = + bbc$. Nam bbc æquatur $\left.\begin{array}{c} bb \\ \underline{c} \end{array}\right|$ & aaa æquatur $\left.\begin{array}{c} aa \\ \underline{a} \end{array}\right|$

Tractatus huius Analytici summaria distributio

In partes duas quarum

- Prima ad exegeticen præparatoria est, in sex capita siue sectiones diuisa, in
  - Prima, *Logistices speciosæ* quatuor operationum formæ practicæ exemplis declarantur.
  - Secunda, æquationum canonicarum primariarum ab originalibus suis deriuatio demonstratur, præmissâ originalium è radicibus binomijs generatarum ordinatâ descriptione.
  - Tertia, æquationum canonicarum secundariarum à primarijs reductio per gradus alicuius parodici sublationem, radice supposititiâ immutatâ manente tractatur.
  - Quarta, æquationum canonicarum tam primariarum quam reductitiarum radices explicatoriæ designantur.
  - Quinta, æquationum Communium per canonicarum æquipollentiam radicum numerus determinatur.
  - Sexta, æquationum Communium reductio per gradus alicuius parodici exclusionem & radicis supposititiæ mutationem traditur.
- Secunda ipsam Exegetices numerosæ praxim continet regulis & exemplis explicatam, ad quam vt ad principale artis magisterium quæcunque in priore parte tractantur subordinata intelligenda sunt.

Logistices

Logistices speciosæ quatuor operationum formæ exemplificatæ.

Additionis exempla.

Addenda	a	Addenda	aa	Addenda	aaa
	b		bc		bcc
Summa	$a+b$	Summa	$aa+bc$	Summa	$aaa+bcc$

Addenda	$a+b$	Addenda	$a+b$
	$c+d$		$c-d$
Summa	$a+b+c+d$	Summa	$a+b+c-d$

Addenda	$a+b$	Addenda	$a+b$
	$-d$		$-b$
Summa	$a+b-d$	Summa	a

Addenda	$a+b$	Addenda	$aa+cc$
	$c+b$		$aa+cc$
Summa	$a+c+2.b$	Summa	$2.aa+2.cc$

Addenda	$aaa+cdf-ddd$	Addenda	$b+7a$
	$aaa+bdd+ddd$		$+9.a$
Summa	$2.aaa+cdf+bdd$	Summa	$b+16a$

Addenda	$b+7.a$	Addenda	$b+9.a$	Addenda	$b-9.a$
	$-9.a$		$b-7.a$		$b+7.a$
Summa	$b-2.a$	Summa	$2.b+2.a$	Summa	$2.b-2.a$

Subductionis exempla.

Positum	a	Positum	aa	Positum	aaa
Subducendum	b	Subducendum	bc	Subducendum	bcc
Residuum	$a-b$	Residuum	$aa-bc$	Residuum	$aaa-bcc$

Positum . . . . $a + b$
Subducendum . . . . $a + d$

Residuum $b - d$

Positum . . . . $a + b$
Subducendum . . . . . . $c - d$

Residuum $a + b - c + d$

Positum . . . . $a + b$
Subducendum . . . . $- d$

Residuum $a + b + d$

Positum . . . . $a + b$
Subducendum . . . . $- b$

Residuum $a + 2. b$

Positum . . . . $a + b$
Subducendum . . . . $c + b$

Residuum $a - c$

Positum . . . . $aa + cc$
Subducendum . . . . $aa + cc$

Residuum 0

Positum . . . . $aaa + cdf - ddd$
Subducendum . . . . . $aaa + bdd + ddd$

Residuum $cdf - bdd - 2. ddd$

Positum . . . . $b + 7. a$
Subducendum . . . . . $+ 9. a$

Residuum $b - 2. a$

Positum . . . . $b + 7. a$
Subducendum . . . . . $- 9. a$

Residuum $b + 16. a$

Positum . . . . $b + 9. a$
Subducendum . . . . . . $b - 7. a$

Residuum $+ 16. a$

Positum . . . . $b - 9. a$
Subducendum . . . . $b + 7. a$

Residuum $- 16. a$

Multiplicationis Exempla.

Multiplicanda . . . . a / b

Factum ab

Multiplicanda . . . . bc / d

Factum bcd

Multiplicanda . . . . aa / bb

Factum $aabb$

Multiplicanda . . . . bbb / bb

Factum $bbbbb$

Multiplicanda . . . . $bbcc$ / dd

Factum $bbccdd$

Multiplicanda . . . . . . $b + a$ / $b + a$

$bb + ba$
$+ ba + aa$

Factum $bb + 2. ba + aa$

Multi-

Multiplicanda	$b - a$
	$b - a$
	$bb - ba$
	$- ba + aa$
Factum	$bb - 2.ba + aa$

Multiplicanda	$b + a$
	$b - a$
	$bb + ba$
	$- ba - aa$
Factum	$bb - aa$

Multiplicanda	$b + c + d$
	a
Factum	$ba + ca + da$

Multiplicanda	$b + c - d$
	$b - c + d$
	$bb + bc - bd$
	$- bc - cc + dc$
	$+ bd + dc - dd$
Factum	$bb - cc + 2.cd - dd$

Diuisionis seu applicationis exempla.

Diuidendum	$bbcc$
Diuisor	cc
Quotiens	bb

Diuidendum	$bcdc$
Diuisor	bdc
Quotiens	c

Diuidendum	$bcdf$
Diuisor	cf
Quotiens	bd

Diuidendum	$ba + ca + da$
Diuisor	a
Quotiens	$b + c + d$

Diuidendum	$ba + ca + da$
Diuisor	$b + c + d$
Quotiens	a

Diuidendum	$bb + 2.ba + aa$
Diuisor	$b + a$
Quotiens	$b + a$

Diuidendum	$bb - aa$
Diuisor	$b - a$
Quotiens	$b + a$

Diuidendum	$bb - aa$
Diuisor	$b + a$
Quotiens	$b - a$

Tria hæc vltima exempla manifesta sunt ex præcognita generatione.

Nota.

Si diuisionis operatio sub forma applicationis concipiatur, pro vocabulis Diuidendum, Diuisor, & Quotiens, adhiberi possunt Applicatum, Metiens, & Ortiuum vel his similia, vt conceptioni magis consona.

Comparationis signa in sequentibus vsurpanda.

AEqualitatis $=$ *vt* $a = b$. *significet* a *æqualem ipsi* b.

Maioritatis $>$ *vt* $a > b$. *significet* a *maiorem quam* b.

Minoritatis $<$ *vt* $a < b$ *significet* a *minorem quam* b.

Fractiones reducibiles reductitijs suis æquatæ.

$\frac{ba}{b} = a.$ | $\frac{bca}{b} = ca.$ | $\frac{bca}{c} = ba$ | $\frac{bcda}{ca} = bd$

$\frac{ba}{c} + d = \frac{ba}{c} + \frac{dc}{c} = \frac{ba+dc}{c}$ | $\frac{ae}{b} + d = \frac{ae+db}{b}$

$\frac{ac}{b} + \frac{dd}{g} = \frac{acg}{bg} + \frac{bdd}{bg} = \frac{acg+bdd}{bg}.$

$\frac{ac}{b} - d = \frac{ac}{b} - \frac{db}{b} = \frac{ac-db}{b}$

$\frac{ac}{b} - \frac{dd}{g} = \frac{acg}{bg} - \frac{ddb}{bg} = \frac{acg-ddb}{bg}$

$\left.\begin{array}{c}\frac{ac}{b}\\ b\end{array}\right| = \frac{acb}{b} = ac.$ | $\left.\begin{array}{c}\frac{ac}{b}\\ d\end{array}\right| = \frac{acd}{b}$ | $\left.\begin{array}{c}\frac{ac}{b}\\ \frac{dd}{g}\end{array}\right| = \frac{acdd}{bg}$

$\cfrac{\frac{aaa}{b}}{d} = \frac{aaa}{bd}$ | $\cfrac{bg}{\frac{ac}{d}} = \frac{bgd}{ac}$ | $\cfrac{\frac{bbb}{c}}{\frac{aaa}{dg}} = \frac{bbbdg}{caaa}$

Æquatio-

Æquationum irregularium ad formam legitimam reductiones exemplificatæ:

Per Antithesim siue particularium transpositionem, quæ fit per communem additionem.

Sit . . . . $aa - dc = gg$ . . . *æquatio reducenda.*
Addatur utrinque . . . . $+ dc$
Hinc fit . . . . $aa = gg + dc$ . . . . *reducta.*

Item sit . . . . $aa - dc = gg - ba$ . . . . *reducenda.*
Addatur utrinque . . . . $+ ba + dc$
Fit inde . . . . $aa + ba = gg + dc$ . . . . . *reducta.*

Per communem diuisionem, quà homogeneum datum à componente graduali liberatur, quæ est Vietæ *Hypobibasmus.*

Sit . . . . $aaa + baa = dca$ . . . . *reducenda.*
Ergo . . . . $\frac{aaa}{a} + \frac{baa}{a} = \frac{dca}{a}$
Ergo . . . . $aa + ba = dc$ . . . . *reducta.*

Per communem diuisionem, quà potestas à componente subgraduali liberatur, quæ est Vietæ *Parabolismus.*

Sit . . . . $baa + dca = gcd$ . . . *reducendà.*
Ergo . . . . $\frac{baa}{b} + \frac{dca}{b} = \frac{gcd}{b}$
Ergo . . . . $aa + \frac{dca}{b} = \frac{gcd}{b}$ . . . *reducta.*

Vel sit . . . . . $baa + dba = cbd$ . . . *reducenda.*
Ergo . . . . . $\frac{baa}{b} + \frac{dba}{b} = \frac{cbd}{b}$
Ergo . . . . . $aa + da = cd$ . . . . *reducta.*

Æquationum

SECTIO SECVNDA.

Æquationum Canonicarum ab originalibus suis deriuatio siue deductio:

Præmissâ ipsarum originalium è radicibus binomijs per genesim constitutarum ordinatâ hac descriptione.

Quadraticæ.

1. $$\begin{array}{l|l} a-b & = aa - ba \\ a-c & \quad - ca + bc \end{array}$$

2. $$\begin{array}{l|l} a-b & = aa - ba \\ a+c & \quad + ca - bc \end{array}$$

3. $$\begin{array}{l|l} a+b & = aa + ba \\ a+c & \quad + ca + bc \end{array}$$

Cubicæ.

1. $$\begin{array}{l|l} a-b & = aaa - baa + bca \\ a-c & \quad - caa + bda \\ a-d & \quad - daa + cda - bcd \end{array}$$

2. $$\begin{array}{l|l} a-b & = aaa - baa + bca \\ a-c & \quad - caa - bda \\ a+d & \quad + daa - cda + bcd \end{array}$$

3. $$\begin{array}{l|l} a+b & = aaa + baa + bca \\ a+c & \quad + caa - bda \\ a-d & \quad - daa - cda - bcd \end{array}$$

4. $$\begin{array}{l|l} a+b & = aaa + baa + bca \\ a+c & \quad + caa + bda \\ a+d & \quad + daa + cda + bcd \end{array}$$

Reciprocæ Cubicarum.

5. $$\begin{array}{l|l} aa-bc & = aaa - daa - bca + bcd. \\ a-d & \end{array}$$

6. $$\begin{array}{l|l} aa+bc & = aaa - daa + bca - bcd \\ a-d & \end{array}$$

7. . . . $\left.\begin{array}{l} aa-bc \\ a+d \\ \hline \end{array}\right| = aaa+daa-bca-bcd$

8. . . . $\left.\begin{array}{l} aa+bc \\ a+d \\ \hline \end{array}\right| = aaa+daa+bca+bcd.$

Tres species Cubicæ æquationum canonicarum ex radicibus æquatis originaliter constitutarum.

Sit . . . $r-a = q.$

9. . . *Ergo* . . $\left.\begin{array}{l} r-a \\ r-a \\ r-a \\ \hline \end{array}\right| = rrr-3.rra+3.raa-aaa = +qqq.$

Sit . . . $r+a = q.$

10. . . *Ergo* . . $\left.\begin{array}{l} r+a \\ r+a \\ r+a \\ \hline \end{array}\right| = rrr+3.rra+3.raa+aaa = +qqq.$

Sit . . . $a-r = q.$

11 . . . *Ergo* . . $\left.\begin{array}{l} a-r \\ a-r \\ a-a \\ \hline \end{array}\right| = aaa-3.raa+3.rra-rrr = +qqq.$

Biquadraticæ.

1. . . . $\left.\begin{array}{l} a-b \\ a-c \\ a-d \\ a-f \\ \hline \end{array}\right| = \begin{array}{llll} aaaa & -baaa & +bcaa & \\ & -caaa & +bdaa & \\ & -daaa & +cdaa & -bcda \\ & -faaa & +bfaa & -bcfa \\ & & +cfaa & -bdfa \\ & & +dfaa & -cdfa+bcdf \end{array}$

2. . . . $\left.\begin{array}{l} a-b \\ a-c \\ a-d \\ a+f \\ \hline \end{array}\right| = \begin{array}{llll} aaaa & -baaa & +bcaa & \\ & -caaa & +bdaa & \\ & -daaa & +cdaa & -bcda \\ & +faaa & -bfaa & +bcfa \\ & & -cfaa & +bdfa \\ & & -dfaa & +cdfa-bcdf \end{array}$

$$3.\ldots\left.\begin{array}{l} a+b \\ a+c \\ a+d \\ a-f \end{array}\right| = \begin{array}{l} aaaa+baaa+bcaa \\ \quad +caaa+bdaa \\ \quad +daaa+cdaa+bcda \\ \quad -faaa-bfaa-bcfa \\ \qquad\quad -cfaa-bdfa \\ \qquad\quad -dfaa-cdfa-bcdf \end{array}$$

$$4.\ldots\left.\begin{array}{l} a-b \\ a-c \\ a+d \\ a+f \end{array}\right| = \begin{array}{l} aaaa-baaa+bcaa \\ \quad -caaa-bdaa \\ \quad +daaa-cdaa+bcda \\ \quad +faaa-bfaa+bcfa \\ \qquad\quad -cfaa-bdfa \\ \qquad\quad +dfaa-cdfa+bcdf \end{array}$$

$$5.\ldots\left.\begin{array}{l} a+b \\ a+c \\ a+d \\ a+f \end{array}\right| = \begin{array}{l} aaaa+baaa+bcaa \\ \quad +caaa+bdaa \\ \quad +daaa+cdaa+bcda \\ \quad +faaa+bfaa+bcfa \\ \qquad\quad +cfaa+bdfa \\ \qquad\quad +dfaa+cdfa+bcdf \end{array}$$

Reciprocæ biquadraticarum.

$$6.\ldots\left.\begin{array}{l} aaa-cdf \\ a-b \end{array}\right| = aaaa-baaa-cdfa+bcdf$$

$$7.\ldots\left.\begin{array}{l} aaa-cdf \\ a+b \end{array}\right| = aaaa+baaa-cdfa-bcdf$$

$$8.\ldots\left.\begin{array}{l} aaa+cdf \\ a-b \end{array}\right| = aaaa-baaa+cdfa-bcdf$$

$$9.\ldots\left.\begin{array}{l} aaa+cdf \\ a+b \end{array}\right| = aaaa+baaa+cdfa+bcdf$$

Aliæ quædam species biquadraticæ æquationum originalium.

$$10.\ldots\left.\begin{array}{l} b-a \\ c-a \\ df-aa \end{array}\right| = \begin{array}{l} bcdf-bdfa+dfaa+baaa \\ \qquad -cdfa-bcaa+caaa-aaaa \end{array}$$

$$11.\ldots\left.\begin{array}{l} b-a \\ c-a \\ df+aa \end{array}\right| = \begin{array}{l} bcdf-bdfa+dfaa-baaa \\ \qquad -cdfa+bcaa-caaa+aaaa \end{array}$$

12. . . $$\left.\begin{array}{r} b+a \\ c+a \\ df-aa \end{array}\right| = \begin{array}{l} bcdf+bdfa+dfaa-baaa \\ \quad +cdfa-bcaa-caaa-aaaa \end{array}$$

13. . . $$\left.\begin{array}{r} b-a \\ c+a \\ df+aa \end{array}\right| = \begin{array}{l} bcdf+bdfa-dfaa+baaa \\ \quad -cdfa+bcaa-caaa-aaaa \end{array}$$

14. . . $$\left.\begin{array}{r} b+a \\ c-a \\ df-aa \end{array}\right| = \begin{array}{l} bcdf-bdfa-dfaa+baaa \\ \quad +cdfa-bcaa-caaa+aaaa \end{array}$$

15. . . . $$\left.\begin{array}{r} b+a \\ c+a \\ df+aa \end{array}\right| = \begin{array}{l} bcdf+bdfa+dfaa+baaa \\ \quad +cdfa+bcaa+caaa+aaaa \end{array}$$

16. . . $$\left.\begin{array}{r} bc-aa \\ df-aa \end{array}\right| = \begin{array}{l} bcdf-dfaa \\ \quad -bcaa+aaaa \end{array}$$

17. . . $$\left.\begin{array}{r} bc-aa \\ df+aa \end{array}\right| = \begin{array}{l} bcdf-dfaa \\ \quad +bcaa-aaaa \end{array}$$

18. . . $$\left.\begin{array}{r} bc+aa \\ df+aa \end{array}\right| = \begin{array}{l} bcdf+dfaa \\ \quad +bcaa+aaaa \end{array}$$

Canoni-

Ergo . . . $aa - ba$

$$ca = -bc$$ quæ est æquatio proposita.

Æquatio igitur canonica proposita ab originali designata. Posito b. vel c. ipsi a. æquali deducitur. Vt est enunciatum.

Canonicarum cubici ordinis deriuatio.

Propositio 3.

Æquatio canonica $$\begin{array}{l} aaa - baa - bca \\ + caa - bda \\ + daa + cda = + bcd. \end{array}$$ ab originali

$$\left.\begin{array}{l} a - b \\ a + c \\ a + d \end{array}\right| = \begin{array}{l} aaa - baa - bca \\ + caa - bda \\ + daa + cda - bcd. \end{array}$$ posito b. ipsi a. æquali deducitur.

Nam si ponatur $a = b$. erit $a - b = o$.

Posito igitur $a = b$. est $$\left.\begin{array}{l} a - b \\ a + c \\ a + d \end{array}\right| = o.$$

Est autem ex genesi $$\left.\begin{array}{l} a - b \\ a + c \\ a + d \end{array}\right| = \begin{array}{l} aaa - baa - bca \\ + caa - bda \\ + daa + cda - bcd. \end{array}$$ quæ est æquatio originalis hic designata.

Ergo . . . $$\begin{array}{l} aaa - baa - bca \\ + caa - bda \\ + daa + cda - bcd = o. \end{array}$$

Ergo . . . $$\begin{array}{l} aaa - baa - bca \\ + caa - bda \\ + daa + cda = + bcd. \end{array}$$ quæ est æquatio proposita.

Æquatio igitur canonica proposita ab originali designata, posito b. ipsi a. æquali deducitur. Vt est enunciatum.

Propositio 4.

Æquatio canonica . . $$\begin{array}{l} aaa - baa + bca \\ - caa - bda \\ + daa - cda = - bcd. \end{array}$$ ab originali

$$\left.\begin{array}{l} a - b \\ a - c \\ a + d \end{array}\right| = \begin{array}{l} aaa - baa + bca. \\ - caa - bda \\ + daa - cda + bcd. \end{array}$$ posito b. vel c. radici a. æquali deducitur.

 Nam

Nam si ponatur . . . $b = a$. erit . . . $a - b = 0$.
Vel . . . . . $c = a$. erit . . . $a - c = 0$.

Posito igitur b. vel c. $= a$. erit $\left.\begin{array}{l} a-b \\ a-c \\ a+d \end{array}\right| = 0.$

Est autem ex genesi $\left.\begin{array}{l} a-b \\ a-c \\ a+d \end{array}\right| = \begin{array}{l} aaa - baa + bca \\ \quad - caa - bda \\ \quad + daa - cda + bcd. \end{array}$ quæ est æquatio originalis hic designata.

Ergo . . . $\begin{array}{l} aaa - baa + bca \\ - caa - bda \\ + daa - sda + bcd. = 0. \end{array}$

Ergo . . . $\begin{array}{l} aaa - baa + bca \\ - caa - bda \\ + daa - cda = - bcd. \end{array}$ quæ est æquatio proposita.

Æquatio igitur canonica proposita ab originali designata, posito b. vel c. ipsi a. æquali deducitur. Vt est enunciatum.

Propositio 5.

Æquatio canonica . . . $\begin{array}{l} aaa - baa - bca. \\ - caa - bda. \\ - daa - cda = + bcd. \end{array}$ ab originali

$\left.\begin{array}{l} a-b \\ a-c \\ a-d \end{array}\right| = \begin{array}{l} aaa - baa - bca \\ \quad - caa - bda \\ \quad - daa - cda - bcd. \end{array}$ posito b. vel c. vel d. ipsi a. æquali deducitur.

Nam si ponatur . . . $b = a$. erit $a - b = 0$.
vel . . . . . . $c = a$. erit $a - c = 0$.
vel . . . . . $d = a$ erit $a - d = 0$.

Posito igitur b. vel c. vel d. $= a$. est $\left.\begin{array}{l} a-b \\ a-c \\ a-d \end{array}\right| = 0.$

Est autem ex genesi $\left.\begin{array}{l} a-b \\ a-c \\ a-d \end{array}\right| = \begin{array}{l} aaa - baa - bca \\ \quad - caa - bda \\ \quad - daa - cda - bcd. \end{array}$ quæ est æquatio originalis hic designata.

Ergo . . . $\begin{array}{l} aaa - baa - bca \\ - caa - bda \\ - daa - cda - bcd = 0. \end{array}$

Ergo

Ergo . . . $aaa - baa - bca$
$\quad - caa - bda$
$\quad - daa - cda = + bcd$. quæ est æquatio canonica proposita.

Aequatio igitur canonica proposita ab originali designata, posito b. vel c. vel d. ipsi a. æquali deducitur. Vt est enunciatum.

Reciprocarum Cubici ordinis deriuatio.

PROPOSITIO 6.

Æquatio reciproca . . . $aaa - baa + cda = + bcd$. ab originali $\left.\begin{array}{c} a-b \\ aa+cd \\ \hline \end{array}\right| = aaa - baa + cda - bcd$. posito b. ipsi a. æquali deriuata est.

Nam si ponatur $b = a$. erit $a - b = o$.

Posito igitur $b = a$ est $\left.\begin{array}{c} a-b \\ aa+cd \\ \hline \end{array}\right| = o$.

Est autem ex genesi $\left.\begin{array}{c} a-b \\ \hline aa+cd \end{array}\right| = aaa - baa + cda - bcd$. quæ est æquatio originalis hic designata.

Ergo . . . $aaa - baa + cda - bcd = o$.

Ergo . . . $aaa - baa + cda = + bcd$. quæ est æquatio reciproca proposita.

Deriuata est igitur æquatio reciproca proposita ab originali designata, posito b. ipsi a æquali. Vt est enunciatum.

PROPOSITIO 7.

Æquatio reciproca . . . $aaa + baa - cda = + bcd$. ab originali $\left.\begin{array}{c} a+b \\ aa-cd \\ \hline \end{array}\right| = aaa + baa - cda - bda$. posito $cd = aa$. deriuata est.

Nam si ponatur $cd = aa$. erit $aa - cd = o$.

Posito igitur . . $cd = aa$. est $\left.\begin{array}{c} a+b \\ aa-cd \\ \hline \end{array}\right| = o$.

Est autem ex genesi $\left.\begin{array}{c} a+b \\ aa-cd \end{array}\right| = aaa + baa - cda - bcd$. quæ est æquatio originalis hic designata.

Ergo

Ergo . . . $aaa + baa - cda - bcd = 0$.

Ergo . . . $aaa + baa - cda = + bcd$. quæ est æquatio proposita.

Deriuata est igitur æquatio reciproca proposita ab originali designata, Posito $cd = aa$. Vt est enunciatum.

Propositio 8.

Æquatio reciproca . . . $aaa - baa - cda = - bcd$. ab originali $\begin{array}{l|} a - b \\ aa - cd \end{array} = aaa - baa - cda + bcd$. posito $b = a$. vel $cd = aa$ deriuata est.

Nam si ponatur $b = a$. erit $a - b = 0$.
vel . . . $cd = aa$. erit $aa - cd = 0$.

Posito igitur $b = a$. vel $cd = aa$. est $\begin{array}{l|} a - b \\ aa - cd \end{array} = 0$.

Est autem ex genesi $\begin{array}{l|} a - b \\ aa - cd \end{array} = aaa - baa - cda + bcd$. quæ est æquatio originalis hic designata.

Ergo . . . . $aaa - baa - cda + bcd = 0$.

Ergo . . . . $aaa - baa - cda = - bcd$. quæ est æquatio reciproca proposita.

Deriuata est igitur æquatio reciproca proposita ab originali designata, posito $b = a$. vel $cd = aa$. Vt est enunciatum.

Canonicarum biquadraticarum deriuatio.

Propositio 9.

Æquatio canonica . . .
$$\begin{array}{llll} aaaa & - baaa & - bcaa & \\ & + caaa & - bdaa & \\ & + daaa & - bfaa & - bcda \\ & + faaa & + cdaa & - bcfa \\ & & + cfaa & - bdfa \\ & & + dfaa & + cdfa = + bcdf. \end{array}$$

ab

ab originali . . .
$$\left.\begin{array}{l} a-b\\ a+c\\ a+d\\ a+f\end{array}\right| = \begin{array}{l} aaaa - baaa - bcaa\\ +caaa - bdaa\\ +daaa - bfaa - bcda\\ +faaa + cdaa - bcfa\\ \qquad +cfaa - bdfa\\ \qquad +dfaa + cdfa - bcdf\end{array}$$

posito $b = a$. deriuata est.

Nam si ponatur $b = a$. erit $a - b = 0$.

Posito igitur $b = a$. est
$$\left.\begin{array}{l} a-b\\ a+c\\ a+d\\ a+f\end{array}\right| = 0.$$

Est autem ex genesi
$$\left.\begin{array}{l} a-b\\ a+c\\ a+d\\ a+f\end{array}\right| = \begin{array}{l} aaaa - baaa - bcaa\\ +caaa - bdaa\\ +daaa - bfaa - bcda\\ +faaa + cdaa - bcfa\\ \qquad +cfaa - bdfa\\ \qquad +dfaa + cdfa - bcdf.\end{array}$$
quæ est æquatio originalis hic designata.

Ergo . . . .
$$\begin{array}{l} aaaa - baaa - bcaa\\ +caaa - bdaa\\ +daaa - bfaa - bcda\\ +faaa + cdaa - bcfa\\ \qquad +cfaa - bdfa\\ \qquad +dfaa + cdfa - bcdf = 0.\end{array}$$

Ergo . . .
$$\begin{array}{l} aaaa - baaa - bcaa\\ +caaa - bdaa\\ +daaa - bfaa - bcda\\ +faaa + cdaa - bcfa\\ \qquad +cfaa - bdfa\\ \qquad +dfaa + cdfa = +bcdf.\end{array}$$
quæ est æquatio proposita.

Deriuata est igitur æquatio canonica proposita ab originali designata, posito $b = a$. V est enuntiatum.

PROPOSITIO 10.

Æquatio canonica . . .
$$\begin{array}{llll} aaaa & -baaa & +bcaa & \\ & -caaa & -bdaa & \\ & +daaa & -bfaa & +bcda \\ & +faaa & -cdaa & +bcfa \\ & & -cfaa & -bdfa \\ & & +dfaa & -cdfa = -bcdf. \end{array}$$

ab originali
$$\left.\begin{array}{l} a-b \\ a-c \\ a+d \\ a+f \end{array}\right| = \begin{array}{llll} aaaa & -baaa & +bcaa & \\ & -caaa & -bdaa & \\ & +daaa & -bfaa & +bcda \\ & +faaa & -cdaa & +bcfa \\ & & -cfaa & -bdfa \\ & & +dfaa & -cdfa+bcdf \end{array}$$

posito $b = a.$ vel $c = a.$ deriuata est.

Nam si ponatur $b = a.$ erit $a-b = 0.$
vel . . . . $c = a.$ erit $a-c = 0.$

Posito igitur b vel $c = a.$ est
$$\left.\begin{array}{l} a-b \\ a-c \\ a+d \\ a+f \end{array}\right| = 0.$$

Est autem ex genesi
$$\left.\begin{array}{l} a-b \\ a-c \\ a+d \\ a+f \end{array}\right| = \begin{array}{llll} aaaa & -baaa & +bcaa & \\ & -caaa & -bdaa & \\ & +daaa & -bfaa & +bcda \\ & +faaa & -cdaa & +bcfa \\ & & -cfaa & -bdfa \\ & & +dfaa & -cdfa+bcdf \end{array}$$

quæ est æquatio originalis hic designata.

Ergo . . .
$$\begin{array}{llll} aaaa & -baaa & +bcaa & \\ & -caaa & -bdaa & \\ & +daaa & -bfaa & +bcda \\ & +faaa & -cdaa & +bcfa \\ & & -cfaa & -bdfa \\ & & +dfaa & -cdfa+bcdf = 0. \end{array}$$

Ergo

Ergo . . . $$\begin{array}{l} aaaa - baaa + bcaa \\ \quad - caaa - bdaa \\ \quad + daaa - bfaa + bcda \\ \quad + faaa - cfaa + bcfa \\ \qquad\quad - cdaa - bdfa \\ \qquad\quad + dfaa - cdfa = - bcdf. \end{array}$$ quæ est æquatio proposita.

Deriuata est igitur æquatio canonica proposita ab originali designata, posito b. vel c $= a$. Vt est enuntiatum.

PROPOSITIO II.

Æquatio canonica . : . $$\begin{array}{l} aaaa - baaa + bcaa \\ \quad - caaa + bdaa \\ \quad - daaa + cdaa - bcda \\ \quad + faaa - bfaa + bcfa \\ \qquad\quad - cfaa + bdfa \\ \qquad\quad - dfaa + cdfa = + bcdf \end{array}$$ ab

originali $$\left.\begin{array}{l} a-b \\ a-c \\ a-d \\ a+f \end{array}\right| = \begin{array}{l} aaaa - baaa + bcaa \\ \quad - caaa + bdaa \\ \quad - daaa + cdaa - bcda \\ \quad + faaa - bfaa + bcfa \\ \qquad\quad - cfaa + bdfa \\ \qquad\quad - dfaa + cdfa - bcdf \end{array}$$ posito

b. vel c. vel $d = a$. deriuata est.

Nam si ponatur . . . $b = a$. erit $a - b = 0$.
vel . . . . . $c = a$. erit $a - c = 0$.
vel . . . . . $d = a$. erit $a - d = 0$.

Posito igitur b. vel c. vel d. $= a$. est $$\left.\begin{array}{l} a-b \\ a-c \\ a-d \\ a+f \end{array}\right| = 0.$$

Est autem ex genesi $$\left.\begin{array}{l} a-b \\ a-c \\ a-d \\ a+f \end{array}\right| = \begin{array}{l} aaaa - baaa + bcaa \\ \quad - caaa + bdaa \\ \quad - daaa + cdaa - bcda \\ \quad + faaa - bfaa + bcfa \\ \qquad\quad - cfaa + bdfa \\ \qquad\quad - dfaa + cdfa = bcdf. \end{array}$$ quæ

est æquatio originalis hic designata.

Ergo

Ergo . . . $\begin{array}{l} aaaa - baaa + bcaa \\ \quad - caaa + bdaa \\ \quad - daaa + cdaa - bcda \\ \quad + faaa - bfaa + bcfa \\ \qquad\qquad - cfaa + bdfa \\ \qquad\qquad - dfaa + cdfa - bcdf = 0 \end{array}$

Ergo . . . $\begin{array}{l} aaaa - baaa + bcaa \\ \quad - caaa + bdaa \\ \quad - daaa + cdaa - bcda \\ \quad + faaa - bfaa + bcfa \\ \qquad\qquad - cfaa + bdfa \\ \qquad\qquad - dfaa + cdfa = + bcdf. \end{array}$ quæ est æquatio proposita.

Deriuata est igitur æquatio canonica proposita ab originali designata, posito b. vel c. vel $d = a$. Vt est enunciatum.

PROPOSITIO 12.

Æquatio canonica . . . . $\begin{array}{l} aaaa - baaa + bcaa \\ \quad - caaa + bdaa \\ \quad - daaa + bfaa - bcda \\ \quad - faaa + cdaa - bcfa \\ \qquad\qquad + cfaa - bdfa \\ \qquad\qquad + dfaa - cdfa = - bcdf \end{array}$

ab originali $\left.\begin{array}{l} a - b \\ a - c \\ a - d \\ a - f \end{array}\right| = \begin{array}{l} aaaa - baaa + bcaa \\ \quad - caaa + bdaa \\ \quad - daaa + bfaa - bcda \\ \quad - faaa + cdaa - bcfa \\ \qquad\qquad + cfaa - bdfa \\ \qquad\qquad + dfaa - cdfa + bcdf \end{array}$ posito b. vel c. vel d. vel $f = a$. deriuata est.

Nam si ponatur . . . $b = a$. erit . . . $a - b = 0$.
Vel . . . . . $c = a$. erit . . . $a - c = 0$.
Vel . . . . $d = a$. erit . . . $a - d = 0$.
Vel . . . . $f = a$. erit . . . $a - f = 0$.

Posito igitur b. vel c. vel d. vel $f = a$. est $\left.\begin{array}{l} a - b \\ a - c \\ a - d \\ a - f \end{array}\right| = 0$.

Est

Est autem ex genesi

$$\begin{array}{l|} a-b \\ a-c \\ a-d \\ a-f \\ \hline \end{array} = \begin{array}{l} aaaa-baaa+bcaa \\ \quad -caaa+bdaa \\ \quad -daaa+bfaa-bcda \\ \quad -faaa+cdaa-bcfa \\ \qquad\quad +cfaa-bdfa \\ \qquad\quad +dfaa-cdfa+bcdf. \end{array}$$

quæ est æquatio originalis hic designata.

Ergo . . .

$$\begin{array}{l} aaaa-baaa+bcaa \\ \quad -caaa+bfaa \\ \quad -daaa+bfaa-bcda \\ \quad -faaa+cdaa-bcfa \\ \qquad\quad +cfaa-bdfa \\ \qquad\quad +dfaa-cdfa+bcdf = 0. \end{array}$$

Ergo . . .

$$\begin{array}{l} aaaa-baaa+bcaa \\ \quad -caaa+bdaa \\ \quad -daaa+bfaa-bcda \\ \quad -faaa+cdaa-bcfa \\ \qquad\quad +cfaa-bdfa \\ \qquad\quad +dfaa-cdfa = -bcdf. \end{array}$$

quæ est æquatio canonica proposita.

Deriuata est igitur æquatio canonica proposita ab originali designata, posito b. vel c. vel d. vel $f = a$. Vt est enunciatum.

Reciprocarum ordinis biquadratici deriuatio.

PROPOSITIO 13.

Æquatio reciproca... $aaaa-baaa+cdfa=+bcdf.$ ab originali $\dfrac{aaa+cdf}{a-b} = aaaa-baaa+cdfa-bcdf.$ posito $b=a.$ deriuata est.

Nam si ponatur $b=a$. erit $a-b=0$.

Posito igitur . . $b=a$. est $\dfrac{aaa+cdf}{a-b}=0$.

Est autem ex genesi $\dfrac{aaa+cdf}{a-b} = aaaa-baaa+cda+cdfa-bcdf$ . . .

quæ est æquatio originalis hic designata.

Ergo . . . $aaaa-baaa+cdfa-bcdf=0$.

Ergo . . . $aaaa-baaa+cdfa=+bcdf$. quæ est æquatio reciproca proposita.

Deriuata est igitur æquatio reciproca proposita ab originali designata, posito $b=a$. Vt est enunciatum.

PROPOSITIO 14.

Æquatio reciproca ... $aaaa + baaa - cdfa = + bcdf$. ab orignali $\frac{aaa - cdf}{a + b} = aaaa + baaa - cdfa - bcdf$. posito $cdf = aaa$. deriuata est.

Nam si ponatur $cdf = aaa$. erit $aaa - cdf = 0$.

Posito igitur $cdf = aaa$. est $\frac{aaa - cdf}{a + b} = 0$.

Est autem ex genesi $\frac{aaa - cdf}{a + b} = aaaa + baaa - cdfa - bcdf$. quæ est æquatio originalis hic designata.

Ergo ... $aaaa + baaa - cdfa - bcdf = 0$.

Ergo ... $aaaa + baaa - cdfa = + bcdf$. quæ est æquatio reciproca proposita.

Deriuata est igitur æquatio reciproca proposita ab originali designata, posito $cdf = aaa$. Vt est enunciatum.

PROPOSITIO 15.

Æquatio reciproca .. $aaaa - baaa - cdfa = - bcdf$. ab originali $\frac{aaa - cdf}{a - b} = aaaa - baaa - cdfa + bcdf$. posito $cdf = aaa$. vel $b = a$. deriuata est.

Nam si ponatur $b = a$. erit $a - b = 0$.
vel .... $cdf = aaa$. erit $aaa - cdf = 0$.

Posito igitur $b = a$. vel $cdf = aaa$. est $\frac{aaa - cdf}{a - b} = 0$.

Est autem ex genesi $\frac{aaa - cdf}{a - b} = aaaa - baaa - cdfa + bcdf$. quæ est æquatio originalis hic designata.

Ergo ... $aaaa - baaa - cdfa + bcdf = 0$.
Ergo ... $aaaa - baaa - cdfa = - bcdf$. quæ est æquatio reciproca proposita.

Deriuata est igitur æquatio reciproca proposita ab originali designata, posito $b = a$. vel $cdf = aaa$. Vt est enuntiatum.

Nota.

Nota.

Æquationum biquadratici ordinis canonicarum deriuatio ab octo originalium speciebus vltimò descriptis, scilicet 10. 11. 12. 13. 14. 15. 16. 17. superiorum exemplo satis manifesta est.

Canonicarum vero deriuationes ab his originalibus, scilicet, 3. Quadratica: 4. & 8. Cubica: 5. 9. & 10. Biquadratica, cum absque radicum priuatiuarum suppositione fieri nequeant, tanquam inutiles negliguntur.

Præterea tres illæ cubici generis æquationes speciales ex radicibus æquatis generatæ, relictâ formali radicum ordinatione, generationis symbolo, pro canonicis deriuatis huc referri possunt. *Videlicet.*

Sit . . . . $b - a \;=\!=\!=\; c.$

Ergo . . . . $aaa - 3.baa + 3.bba \;=\!=\!=\; -ccc + bbb$

Sit . . . . . . $a + b \;=\!=\!=\; c.$

Ergo . . . . . $aaa + 3.baa + 3.bba \;=\!=\!=\; +ccc - bbb$

Sit . . . . . . $a - b \;=\!=\!=\; c.$

Ergo . . . . . . $aaa - 3.baa + 3.bba \;=\!=\!=\; +ccc + bbb.$

Æquationum Canonicarum quarum deriuationes in secunda hac Sectione demonstrantur, recollectio.

Quadraticæ.

1. . . . . . $\begin{array}{l} aa - ba \\ +ca \;=\!=\!=\; +bc \end{array}$

2. . . . . . $\begin{array}{l} aa - ba \\ -ca \;=\!=\!=\; -bc \end{array}$

Cubicæ.

1. . . . . $\begin{array}{l} aaa - baa - bca \\ +caa - bda \\ +daa + cda \;=\!=\!=\; +bcd \end{array}$

2. . . . . $\begin{array}{l} aaa - baa + bca \\ -caa - bda \\ +daa - cda \;=\!=\!=\; -bcd \end{array}$

3. . . . . $\begin{array}{l} aaa - baa - bca \\ -caa - bda \\ -daa - cda \;=\!=\!=\; +bcd \end{array}$

4. . . . . $aaa - baa + cda \;=\!=\!=\; +bcd$

5. . . . . $aaa + baa - cda \;=\!=\!=\; +bcd$

6. . . . . $aaa - baa - cda \;=\!=\!=\; -bcd$

Biqua-

Biquadraticæ.

1\. . . . .
$$\begin{array}{llll} aaaa - baaa - bcaa & & & \\ +\,caaa - bdaa & & & \\ +\,daaa - bfaa - bcda & & & \\ +\,faaa + cdaa - bcfa & & & \\ \quad\; + cfaa - bdfa & & & \\ \quad\; + dfaa + cdfa & = & + bcdf & \end{array}$$

2\. . . . .
$$\begin{array}{llll} aaaa - baaa + bcaa & & & \\ -\,caaa - bdaa & & & \\ +\,daaa - bfaa + bcda & & & \\ +\,faaa - cdaa + bcfa & & & \\ \quad\; - cfaa - bdfa & & & \\ \quad\; + dfaa - cdfa & = & - bcdf & \end{array}$$

3\. . . . .
$$\begin{array}{llll} aaaa - baaa + bcaa & & & \\ -\,caaa + bdaa & & & \\ -\,daaa + cdaa - bcda & & & \\ +\,faaa - bfaa + bcfa & & & \\ \quad\; - cfaa + bdfa & & & \\ \quad\; - dfaa + cdfa & = & + bcdf & \end{array}$$

4\. . . . .
$$\begin{array}{llll} aaaa - baaa + bcaa & & & \\ -\,caaa + bdaa & & & \\ -\,daaa + bfaa - bcda & & & \\ -\,faaa + cdaa - bcfa & & & \\ \quad\; + cfaa - bdfa & & & \\ \quad\; + dfaa - cdfa & = & - bcdf & \end{array}$$

5\. . . . . $aaaa - baaa + cdfa = + bcdf$

6\. . . . . $aaaa + baaa - cdfa = + bcdf$

7\. . . . . $aaaa - baaa - cdfa = - bcdf$

Æquatio-

Æquationum canonicarum secundariarum à primarijs *reductio per gradus alicuius parodici sublationem radice supposititiâ inuariatâ manente.*

Æquationis canonicæ quadraticæ reductio singularis.

PROBLEMA 1.

Æquationem binomiam $\begin{matrix} aa - ba \\ + ca \end{matrix} = + bc$ ad solinomiã $aa = bb$. reducere, sublato scilicet gradu primo a.

Ponatur $b = c$.

Et in æquatione binomia reducenda fiat mutatio c in b.

Erit inde . . . . . $\begin{matrix} aa - ba \\ + ba \end{matrix} = + bb.$

Ergo sublatis particularibus ex contradictione redundantibus,
fit . . . . . $aa = bb.$ quæ est æquatio solinomia requisita.

Sic igitur facta est æquationis propositæ binomiæ ad solinomiam requisitam reductio imperata.

Canonicarum cubici ordinis reductiones.

PROBLEMA 2.

Æquationem trinomiam $\begin{matrix} aaa - baa + bca \\ - caa - bda \\ + daa - cda \end{matrix} = - bcd$ ad binomiam $\begin{matrix} aaa - bba \\ - bca \\ - cca \end{matrix} = \begin{matrix} - bbc \\ - bcc. \end{matrix}$ reducere, sublato scilicet gradu secundo aa.

Ponatur $b + c = d$.

Et in æquatione trinomia reducenda fiat mutatio d in $b + c$.

 Erit

Erit inde . . . .
$$\begin{array}{l} aaa - baa + bca \\ - caa - bba \\ + baa - bca \\ + caa - bca \\ - cca = - bbc \\ \qquad\qquad - bcc. \end{array}$$

Ergo reiectis particularibus ex contradictione redundantibus fit . . . .
$$\begin{array}{l} aaa - bba \\ - bca \\ - cca = - bbc \\ \qquad\qquad - bcc. \end{array}$$
Quæ est æquatio binomia requisita.

Sic igitur facta est æquationis trinomiæ propositæ ad requisitam binomiam reductio imperata.

PROBLEMA. 3.

Æquationem trinomiam
$$\begin{array}{l} aaa - baa + bca \\ - caa - bda \\ + daa - cda = - bcd. \end{array}$$
sublato gradu primo a. ad binomiam
$$\frac{\begin{array}{l} aaa - bbaa \\ - bcaa \\ - ccaa \end{array}}{b+c} = \frac{- bccc}{b+c}.$$
reducere.

Ponatur $bc = bd + cd$

Hinc in æquatione proposita per particularium contradictionem eliditur gradus primus a.

Vnde restabit . . .
$$\begin{array}{l} aaa - baa \\ - caa \\ + daa = - bcd. \end{array}$$
æquationis pars adhuc reducenda.

Posito $bc = bd + cd$. est $\frac{bc}{b+c} = d$.

Fiat igitur in parte æquationis restante mutatio d. in $\frac{bc}{b+c}$

Hinc erit
$$\begin{array}{l} aaa - baa \\ - caa \\ + \frac{bcaa}{b+c} = - \frac{bbcc}{b+c} \end{array}$$

Reducantur reliqua baa. & caa. ad communem diuisorem $b + c$.

Erit inde . . . .
$$\begin{array}{l} aaa - bbaa \\ - bcaa \\ - bcaa \\ - ccaa \\ + \frac{bcaa}{b+c} = - \frac{bbcc}{b+c} \end{array}$$

Tollantur

Tollantur contradictoria redundantia.

Sic demum fit . . . . $\frac{aaa - bbaa - bcaa - ccaa}{b+c} = \frac{-bbcc.}{b+c}$ æquatio binomia reductioni præscripta.

Atque sic facta est æquationis propositæ ad præscriptam reductio imperara.

PROBLEMA 4.

Æquationem trinomiam . . . . $aaa + baa + bca + caa - bda - daa - cda = + bcd$

ad binomiam $aaa - bba - bca - cca = + bbc + bcc.$ reducere, sublato scilicet gradu secundo aa.

Ponatur $b + c = d.$

Et in æquatione trinomia reducenda fiat mutatio $d.$ in $b + c.$

Erit inde . . . . $aaa + baa + bca + caa - bba - baa - bca - caa - bca - cca = + bbc + bcc$

Ergo sublatis particularibus ex contradictione redundantibus fit . . . . . $aaa - bba - bca - cca = + bbc + bcc.$ Quæ est æquatio binomia requisita.

Sic igitur facta est æquationis propositæ trinomiæ ad binomiam requisitam reductio imperata.

PRO-

PROBEMA 5.

Æquationem trinomiam . . . $aaa + baa + bca$
$+ caa - bda$
$- daa - cda = + bcd.$

ad binomiam . . . $aaa + bbaa$
$+ bcaa$
$+ \frac{ccaa}{b+c} = + \frac{bbcc}{b+c}$. reducere, ſublato ſcil. gradu primo a.

Ponatur . . . . $bc = bd + cd.$

Hinc in æquatione reducenda per particularium contradictionem tollitur gradus primus a.

Vnde reſtabit . . . $+ baa$
$+ caa$
$- daa = + bcd.$ æquationis pars adhuc reducenda.

Ex ſuppoſito $bc = bd + cd.$ eſt $\frac{bc}{b+c} = d$

In parte igitur illa æquationis reſtante & in particularibus quibus $d.$ ineſt fiat mutatio $d.$ in $\frac{bc}{b+c}$

Erit inde . . . . $aaa + baa$
$+ caa$
$- \frac{bcaa}{b+c} = + \frac{bbcc}{b+c}$

Reducantur particularia reliqua $baa.$ $caa.$ ad communem diuiſorem. $b + c.$

Hinc erit . . . . $aaa + bbaa$
$+ bcaa$
$+ bcaa$
$+ ccaa$
$- \frac{bcaa}{b+c} = + \frac{bbcc}{b+c}$

Reijciantur contradictoria redundantia.

Hinc fit . . . . $aaa + bbaa$
$+ bcaa$
$+ \frac{ccaa}{b+c} = + \frac{bbcc}{b+c}.$ Quæ eſt æquatio binomia requiſita.

Atque ſic facta eſt æquationis trinomiæ propoſitæ ad requiſitam binomiam reductio imperata.

PRO-

PROBLEMA 6.

Æquationem trinomiam ... $aaa - 3.baa + 3.bba = bbb - ccc$ ad binomiam $aaa + 3.bca = bbb - ccc.$ reducere, sublato scilicet gradu secundo aa.

Resumatur . . . . $b - a = +c.$ generationis radix æquata.

Ergo . . . . $\begin{array}{c|}-a+b\\ \hline 3.ba\end{array} = \begin{array}{c|}+c\\ \hline 3.ba\end{array}$

Sed . . . . $\begin{array}{c|}-a+b\\ \hline 3.ba\end{array} = -3.baa + 3\,bba.$

Et . . . . $\begin{array}{c|}+c\\ \hline 3\,ba\end{array} = +3.bca.$

Ergo . . . . $-3.baa + 3.bba = +3.bca.$

Ergo . . . . $aaa - 3.baa + 3.bba = aaa + 3.bca.$

Sed . . . . $aaa - 3.baa + 3.bba = bbb - ccc.$

Est enim ipsa æquatio trinomia proposita.

Ergo . . . . $aaa + 3.bca = bbb - ccc.$ quæ quidem est æquatio binomia requisita.

Posito igitur $b - a = +c.$ Fit æquationis trinomiæ propositæ ad requisitam binomiam reductio imperata.

PROBLEMA 7.

Æquationem trinomiam ... $aaa + 3.baa + 3.bba = -bbb + ccc.$ ad binomiam $aaa + 3.bca = -bbb + ccc.$ reducere, sublato scilicet gradu secundo aa.

Resumatur . . . . $a + b = +c.$ generationis radix æquata.

Ergo . . . . $\begin{array}{c|}+a+b\\ \hline 3.ba\end{array} = \begin{array}{c|}+c\\ \hline 3.ba\end{array}$

Sed . . . . $\begin{array}{c|}+a+b\\ \hline 3.ba\end{array} = +3.baa + 3.bba.$

Et . . . . $\begin{array}{c|}+c\\ \hline 3.ba\end{array} = +3.bca$

Ergo . . . . $+3.baa + 3.bba = +3.bca.$

Ergo . . . . $aaa + 3.baa + 3\,bba = aaa + 3.bca.$

Sed . . . . $aaa + 3.baa + 3.bba = -bbb + ccc.$

Est enim ipsa æquatio trinomia proposita.

Ergo . . . . $aaa + 3.bca = -bbb + ccc.$ quæ quidem est æquatio binomia requisita.

Posito igitur $a + b = +c.$ fit æquationis trinomiæ propositæ ad requisitam binomiam reductio imperata.

PROBLEMA 8.

Æquationem trinomiam . . . $aaa - 3.baa + 3.bba = +bbb + ccc.$ ad binomiam $aaa - 3.bca = +bbb + ccc.$ reducere, sublato scilicet gradu secundo $aa.$

Resumatur . . . . $a - b = +c.$ generationis radix æquata.

Ergo . . . . $\begin{array}{l|}-a+b\\ \underline{3.ba}\end{array} = \begin{array}{l|}-c\\ \underline{3.ba}\end{array}$

Sed . . . . $\begin{array}{l|}-a+b\\ \underline{3.ba}\end{array} = -3.baa + 3.bba$

Et . . . . $\begin{array}{l|}-c\\ \underline{3.ba}\end{array} = -3.bca$

Ergo . . . . $-3.baa + 3.bba. = -3.bca.$

Ergo . . . . $aaa - 3.baa + 3.bba = aaa - 3.bca.$

Sed . . . . $aaa - 3.baa + 3.bba = +bbb + ccc.$

Est enim ipsa æquatio trinomia proposita.

Ergo . . . . $aaa - 3.bca = +bbb + ccc.$ quæ quidem est æquatio binomia requisita.

Posito igitur $a - b = +c.$ facta est æquationis trinomiæ propositæ ad requisitam binomiam reductio imperata.

Canonicarum biquadratici ordinis reductiones.

PROBLEMA 9.

Æquationẽ quadrinomiã . . $aaaa - baaa + bcaa$

$$\begin{array}{l} aaaa - baaa + bcaa \\ \quad - caaa + bdaa \\ \quad - daaa + cdaa - bcda \\ \quad + faaa - bfaa + bcfa \\ \qquad\qquad - cfaa + bdfa \\ \qquad\qquad - dfaa + cdfa = +bcdf \end{array}$$

ad

ad trinomiam

$$\begin{array}{llll} aaaa & -bbaa & +bbca & \\ & -ccaa & +bbda & \\ & -ddaa & +bcca & \\ & -bcaa & +ccda & \\ & -bdaa & +bdda & \\ & -cdaa & +cdda & \\ & & +2.bcda & = +bbcd \\ & & & \;\;\, +bccd \\ & & & \;\;\, +bcdd. \end{array}$$

reducere, sublato scilicet gradu tertio aaa.

Ponatur $b+c+d = f$.

Et in æquatione quadrinomia quæ reducenda proponitur fiat mutatio f in $b+c+d$.

Erit inde . . . .

$$\begin{array}{llll} aaaa & -baaa & +bcaa & \\ & -caaa & +bdaa & \\ & -daaa & +cdaa & -bcda \\ & +baaa & -bbaa & +bcba \\ & +caaa & -bcaa & +bcca \\ & +daaa & -bdaa & +bcda \\ & & -cbaa & +bdba \\ & & -ccaa & +bdca \\ & & -cdaa & +bdda \\ & & -ddaa & +cdba \\ & & -dcaa & +cdca \\ & & -ddaa & +cdda = +bcdb \\ & & & \qquad\quad\;\, +bcdc \\ & & & \qquad\quad\;\, +bcdd \end{array}$$

Reijciantur particularia ex contradictione redundantia.

Hinc fit . . . .

$$\begin{array}{llll} aaaa & -bbaa & +bbca & \\ & -ccaa & +bbda & \\ & -ddaa & +bcca & \\ & -bcaa & +ccda & \\ & -bdaa & +bdda & \\ & -cdaa & +cdda & \\ & & +2.bcda & = +bbcd \\ & & & \;\;\, +bccd \\ & & & \;\;\, +bcdd. \end{array}$$

Est autem ista æquatio trinomia requisita.

Sic igitur facta est æquationis propositæ ad requisitam reductio imperata.

PRO-

PROBLEMA 10.

Æquationē quadrinomiam . .
$$\begin{array}{llll}
aaaa & -baaa & +bcaa & \\
 & -caaa & +bdaa & \\
 & -daaa & +cdaa & -bcda \\
 & +faaa & +bfaa & +bcfa \\
 & & -cfaa & +bdfa \\
 & & -dfaa & +cdfa = +bcdf.
\end{array}$$

ad trinomiam
$$\begin{array}{llll}
aaaa & -bbaaa & +bbcca & \\
 & -ccaaa & +bbdda & \\
 & -ddaaa & +ccdda & \\
 & -bcaaa & +bcdda & \\
 & -bdaaa & +bccda & \\
 & -cdaaa & +bbcda = & +bbccd \\
 & \overline{b+c+d} & \overline{b+c+d} & +bbcdd \\
 & & & +bccdd. \\
 & & & \overline{b+c+d}
\end{array}$$
reducere, sublato scilicet gradu secundo aa.

Ponatur. : . . . $bc+bd+cd = bf.+cf+df.$

Hinc in æquatione proposita per particularium contradictionem tollitur gradus secundus aa.

Vnde restabit . . . .
$$\begin{array}{lll}
aaaa & -baaa & -bcda \\
 & -caaa & +bcfa \\
 & -daaa & +bdfa \\
 & +faaa & +cdfa = +bcdf.
\end{array}$$
æquationis pars adhuc reducenda.

Ex supposito $bc+bd+cd = bf+cf+df$ est $\frac{bc+bd+cd}{b+c+d} = f.$

In parte igitur æquationis restante & in particularibus quibus $f.$ inest fiat mutatio $f.$ in $\frac{bc+bd+cd}{b+c+d}$

Erit inde . . . .
$$\begin{array}{llll}
aaaa & -baaa & -bcda & \\
 & -caaa & +bbcca & \\
 & -daaa & +bbcda & \\
 & +bcaaa & +bccda & \\
 & +bdaaa & +bbcda & \\
 & +cdaaa & +bbdda & \\
 & \overline{b+c+d} & +bcdda & \\
 & & +bccda & \\
 & & +bcdda & \\
 & & +ccdda = & +bbccd \\
 & & \overline{b+c+d} & +bbcdd \\
 & & & +bccdd \\
 & & & \overline{b+c+d}
\end{array}$$

Redu-

Reducantur particularia reliqua $baaa$. $caaa$. $daaa$. & $bcda$. ad communem diuisorem $b+c+d$.

Hinc erit . . . . $aaaa \begin{array}{l} -bbaaa \\ -bcaaa \\ -bdaaa \\ -bcaaa \\ -ccaaa \\ -dcaaa \\ -bdaaa \\ -cdaaa \\ -ddaaa \\ +bcaaa \\ +bdaaa \\ +cdaaa \\ \hline b+c+d \end{array} \begin{array}{l} -bbcda \\ -bccda \\ -bcdda \\ +bbcca \\ +bbcda \\ +bccda \\ +bbcda \\ +bbdda \\ +bcdda \\ +bccda \\ +bcdda \\ +ccdda \\ \hline b+c+d \end{array} = \begin{array}{l} +bbccd \\ +bbcdd \\ +bccdd \\ \hline b+c+d \end{array}$

Reijciantur particularia ex contradictione redundantia.

Hinc fit . . . . $aaaa \begin{array}{l} -bbaaa \\ -bcaaa \\ -ccaaa \\ -bdaaa \\ -cdaaa \\ -ddaaa \\ \hline b+c+d \end{array} \begin{array}{l} +bbcca \\ +bbcda \\ +bccda \\ +bbdda \\ +bcdda \\ +ccdda \\ \hline b+c+d \end{array} = \begin{array}{l} +bbccd \\ +bbcdd \\ +bccdd \\ \hline b+c+d \end{array}$

Est autem ista æquatio trinomia requisita.

Facta est igitur æquationis quadrinomiæ propositæ ad requisitam trinomiam, reductio imperata.

PROBLEMA II.

Æquationẽ quadrinomiã . . $\begin{array}{llll} aaaa & -baaa & +bcaa & \\ & -caaa & +bdaa & \\ & -daaa & +cdaa & -bcda \\ & +faaa & -bfaa & +bcfa \\ & & -cfaa & +bdfa \\ & & -dfaa & +cdfa = +bcdf \end{array}$

ad trinomiam

$$\begin{aligned} aaaa &- bbcaa \\ - bbdaaa &+ bbccaa \\ - bccaaa &+ bbddaa \\ - bddaaa &+ ccddaa \\ - ccdaaa &+ bcddaa \\ - cddaaa &+ bccdaa \\ - 2.bcdaaa &+ bbcdaa = + bbccdd \end{aligned}$$

$$\overline{bc+bd+cd} \quad \overline{bc+bd+cd} \quad \overline{bc+bd+cd.}$$

reducere, sublato scilicet gradu primo a.

Ponatur. : . . . . $bcd = bcf + bdf + cdf$.

Hinc in æquatione proposita quadrinomia per particularium contradictionem tollitur gradus primus a.

Vnde restabit . . . .

$$\begin{aligned} aaaa - baaa &+ bcaa \\ - caaa &+ bdaa \\ - daaa &+ cdaa \\ + faaa &- bfaa \\ &- cfaa \\ &- dfaa = + bcdf. \end{aligned}$$

æquationis pars adhuc reducenda.

Ex supposito $bcd = bcf + bdf + cdf$ est $\dfrac{bcd}{bc+bd+cd.} = f$.

In parte igitur æquationis restante & in particularibus quibus f. inest fiat primo mutatio f. in $\dfrac{bcd}{bc+bd+cd}$

Secundo particularium reliquorum ad communem diuisorem $bc+bd+cd$. reductio.

Tertio particularium ex contradictione redundantium reiectio.

His peractis (vt in 10. Probl.) fit . . .

$$\begin{aligned} aaaa &- bbcaa \\ - bbdaaa &+ bbccaa \\ - bccaaa &+ bbddaa \\ - bddaaa &+ ccddaa \\ - ccdaaa &+ bcddaa \\ - cddaaa &+ bccdaa \\ - 2.bcdaaa &+ bbcdaa = + bbccdd \end{aligned}$$

$$\overline{bc+bd+cd} \quad \overline{bc+bd+cd} \quad \overline{bc+ba+cd}$$

Est autem ista æquatio trinomia requisita, in qua tollitur gradus primus a.

Et sic perficitur reductio imperata.

PRO-

PROBLEMA 12.

Æquationē quadrinomiā . .

$$\begin{array}{llll} aaaa+baaa & +bcaa & & \\ +caaa & +bdaa & & \\ +daaa & +cdaa & +bcda & \\ -faaa & -bfaa & -bcfa & \\ & -cfaa & -bdfa & \\ & -dfaa & -cdfd & =\!=\!= +bcdf \end{array}$$

ad trinomiam

$$\begin{array}{llll} aaaa-bbaa & -bbca & & \\ -ccaa & -bbda & & \\ -ddaa & -bcca & & \\ -bcaa & -ccda & & \\ -bdaa & -bdda & & \\ -cdaa & -cdda & & \\ & -2.bcda & =\!=\!= & +bbcd \\ & & & +bccd \\ & & & +bcdd. \end{array}$$

reducere, sublat scilicet gradu tertio aaa.

Ponatur . . . . . $b+c+d =\!=\!= f$.
Et in æquatione quadrinomia quæ reducenda proponitur fiat mutatio f. in $b+c+d$.
Erit inde . . . .

$$\begin{array}{lllll} aaaa+baaa & +bcaa & & & \\ +caaa & +bdaa & & & \\ +daaa & +cdaa & +bcda & & \\ -baaa & -bbaa & -bcba & & \\ -caaa & -bcaa & -bcca & & \\ -daaa & -bdaa & -bcda & & \\ & -cbaa & -bdba & & \\ & -ccaa & -bdca & & \\ & -cdaa & -bdda & & \\ & -dbaa & -cdba & & \\ & -dcaa & -cdca & & \\ & -ddaa & -cdda & =\!=\!= & +bcdb \\ & & & & +bcdc \\ & & & & +bcdd \end{array}$$

Reijciantur particularia ex contradictione redundantia.
Hinc fit . . . .

$$\begin{array}{llll} aaaa-bbaa & -bbca & & \\ -bcaa & -bcca & & \\ -ccaa & -bbda & & \\ -dbaa & -bdda & & \\ -dcaa & -ccda & & \\ -ddaa & -cdda & & \\ & -2bcda & =\!=\!= & +bbcd \\ & & & +bcdc \\ & & & +bcdd. \end{array}$$

quæ est æquatio trinomia requisita.
Sic igitur facta est æquationis propositæ ad requisitam reductio imperata.

PRO-

PROBLEMA 13.

Æquationē quadrinomiā . . $\begin{array}{llll} aaaa + baaa & + bcaa & & \\ + caaa & + bdaa & & \\ + daaa & + cdaa & + bcda & \\ - faaa & - bfaa & - bcfa & \\ & - cfaa & - bdfa & \\ & - dfaa & - cdfa & = + bcdf \end{array}$

ad trinomiam $\begin{array}{llll} aaaa + bbaaa & - bbcca & & \\ + ccaaa & - bbdda & & \\ + ddaaa & - ccdda & & \\ + bcaaa & - bcdda & & \\ + bdaaa & - bccda & & \\ + cdaaa & - bbcda & = & + bbccd \\ \overline{b + c + d} & \overline{b + c + d} & & + bbcdd \\ & & & + bccdd \\ & & & \overline{b + c + d}. \end{array}$ reducere, sublato scilicet gradu secundo aa.

Ponatur . . . . $bc + bd + cd = bf + cf + df.$

Hinc in æquatione quadrinomia proposita per particularium contradictionem tollitur gradus secundus aa.

Vnde restabit . . . . $\begin{array}{ll} aaaa + baaa & + bcda \\ + caaa & - bcfa \\ + daaa & - bdfa \\ - faaa & - cdfa = + bcdf. \end{array}$ æquationis pars adhuc reducenda.

Ex supposito $bc + bd + cd = bf + cf + df.$ est $\frac{bc + bd + cd}{b + c + d} = f.$

In parte igitur illa æquationis restante, & in particularibus quibus $f.$ inest, fiat primò mutatio $f.$ in $\frac{bc + bd + cd}{b + c + d}$

Secundò particularium reliquorum ad communem diuisorem reductio.

Tertiò particularium ex contradictione redundantium reiectio.

His peractis (vt in 10. Probl.) fit . . $\begin{array}{llll} aaaa + bbaaa & - bbcca & & \\ + ccaaa & - bbdda & & \\ + ddaaa & - ccdda & & \\ + bcaaa & - bcdda & & \\ + bdaaa & - bccda & & \\ + cdaaa & - bbcda & = & + bbccd \\ \overline{b + c + d} & \overline{b + c + d} & & + bbcdd \\ & & & + bccdd \\ & & & \overline{b + c + d} \end{array}$

Et

Et autem ista æquatio trinomia requisita.

Atque sic perficitur reductio imperata.

PROBLEMA 14.

Æquationẽ quadrinomiã ...
$$\begin{array}{lll} aaaa+baaa & +bcaa & \\ +caaa & +bdaa & \\ +daaa & +cdaa & +bcda \\ -faaa & -bfaa & -bcfa \\ & -cfaa & -bdfa \\ & -dfaa & -cafa = +bcdf \end{array}$$

ad trinomiam
$$\begin{array}{ll} aaaa & +\dfrac{bbcaaa}{} \\ & +bbdaaa+bbccaa \\ & +bccaaa+bbddaa \\ & +ccdaaa+ccddaa \\ & +bddaaa+bcddaa \\ & +cddaaa+bccdaa \\ & +\dfrac{2.bcdaaa}{bc+bd+cd}+\dfrac{bbcdaa}{bc+bd+cd} = +\dfrac{bbccdd.}{bc+bd+cd} \end{array}$$
reducere, sublato scilicet gradu primo a.

Ponatur . . . . $bcd = bcf+bdf+cdf.$

Hinc in æquatione quadrinomia proposita per particularium contradictionem eliditur gradus primus a.

Vnde restabit . . . .
$$\begin{array}{ll} aaaa+baaa & +bcaa \\ +caaa & +bdaa \\ +daaa & +cdaa \\ -faaa & -bbaa \\ & -cfaa \\ & -dfaa = +bcdf. \end{array}$$
æquationis propositæ pars adhuc reducenda.

Ex supposito $bcd = bcf+bdf+cdf.$ est $\dfrac{bcd}{bc+ba+cd} = f.$

In parte igitur illa æquationis restante & in particularibus quibus $f.$ inest, fiat primò mutatio $f.$ in $\dfrac{bcd}{bc+bd+cd}$

Secundò particularium reliquorum ad communem diuisorem reductio.

Tertiò particularium ex contradictione redundantium reiectio.

His peractis (vt in 10. Probl.) fit...

$$\begin{array}{llll} aaaa & +bbcaaa & & \\ & +bbdaaa & +bbccaa & \\ & +bccaaa & +bbddaa & \\ & +ccdaaa & +ccddaa & \\ & +bddaaa & +bcddaa & \\ & +cddaaa & +bccdaa & \\ & +2.bcdaaa & +bbcdaa & = +bbccdd \\ \hline & bc+bd+cd & bc+bd+cd & \quad bc+bd+cd \end{array}$$

quæ est æquatio trinomia requisita in qua tollitur gradus primus a.
Et sic perficitur reductio imperata.

PROBLEMA 15.

Æquationē quadrinomiā..

$$\begin{array}{llll} aaaa & -baaa & +bcaa & \\ & -caaa & -bdaa & \\ & +daaa & -cdaa & +bcda \\ & +faaa & -bfaa & +bcfa \\ & & -cfaa & -bdfa \\ & & +dfaa & -cdfa = -bcdf \end{array}$$

ab trinomiam

$$\begin{array}{llll} aaaa & +bdaa & +bbca & \\ & +cdaa & +bcca & \\ & -bbaa & +bdda & \\ & -bcaa & +cdda & \\ & -ccaa & -bbda & \\ & -ddaa & -ccda & \\ & & -2.bcda & = -bbcd \\ & & & \;\; -bccd \\ & & & \;\; +bcdd. \end{array}$$

reducere, sublato scilicet gradu tertio aaa.

Si sit $b+c = d+f$. erit $b+c-d = f$.
Ponatur igitur $b+c-d = f$.
Et in æquatione quadrinomia quæ reducenda proponitur fiat primo mutatio f. in $b+c-d$.
Secundò reiectio redundantium ex contradictione.

His peractis (vt in 10. Probl.) fit...

$$\begin{array}{llll} aaaa & +bdaa & +bbca & \\ & +cdaa & +bcca & \\ & -bbaa & +bdda & \\ & -bcaa & +cdda & \\ & -ccaa & -bbda & \\ & -ddaa & -ccda & \\ & & -2.bcda & = -bbcd \\ & & & \;\; -bccd \\ & & & \;\; +bcdd \end{array}$$

quæ est æquatio trinomia requisita, in qua tollitur gradus tertius aaa.
Et sic perficitur reductio imperata.

PRO-

PROBLEMA 16.

Æquationẽ quadrinomiã . .
$$\begin{array}{llll} aaaa & -baaa & +bcaa & \\ & -caaa & -bdaa & \\ & +daaa & -cdaa & +bcda \\ & +faaa & -bfaa & +bcfa \\ & & -cfaa & -bdfa \\ & & +dfaa & -cdfa = -bcdf \end{array}$$

ad trinomiam
$$aaaa - \frac{\begin{array}{l} bbaaa \\ -bcaaa \\ -ccaaa \\ -ddaaa \\ +bdaaa \\ +cdaaa \end{array}}{b+c-d} + \frac{\begin{array}{l} bbcca \\ +bbdda \\ +bcdda \\ +ccdda \\ -bbcda \\ -bccda \end{array}}{b+c-d} = \frac{\begin{array}{l} -bbccd \\ +bbcdd \\ +bccdd \end{array}}{b+c-d}$$

reducere, sublato scilicet gradu secundo aa.

Si sit $bc + df = bd + cd + bf + cf$. | hoc est $bc - bd - cd = bf + cf - df$ erit $\frac{bc - bd - cd}{b + c - d} = f$.

Ponatur igitur $\frac{bc - bd - cd}{b + c - d} = f$.

Et in æquatione quadrinomia quæ reducenda proponitur, fiat primo mutatio f. in $\frac{bc - bd - cd}{b + c - d}$

Secundò reductio particularium reliquorum ad communem diuisorem $b + c - d$.

Tertiò reiectio redundantium ex contradictione.

His peractis (vt in 9. Probl) fit . . .
$$aaaa - \frac{\begin{array}{l} bbaaa \\ -bcaaa \\ -ccaaa \\ -ddaaa \\ +bdaaa \\ +cdaaa \end{array}}{b+c-d} + \frac{\begin{array}{l} bbcca \\ +bbdda \\ +bcdda \\ +ccdda \\ -bbcda \\ -bccda \end{array}}{b+c-d} = \frac{\begin{array}{l} -bbccd \\ +bbcdd \\ +bccdd \end{array}}{b+c-d}$$

quæ est æquatio trinomia requisita, in qua tollitur gradus secundus aa.

Et sic perficitur reductio imperata.

PRO-

PROBLEMA 17.

Reducibilis est quoque quadrinomia superiùs proposita ad trinomiam istam

$$aaaa+\frac{\begin{array}{l}+bbaaa\\+bcaaa\\+ccaaa\\+ddaaa\\-bdaaa\\-cdaaa\end{array}}{d-b-c}\;\frac{\begin{array}{l}-bbcca\\-bcdda\\-bbdda\\-ccdda\\+bbcda\\+bccda\end{array}}{d-b-c}=\frac{\begin{array}{l}-bbcdd\\-bccdd\\+bbccd\end{array}}{d-b-c}$$

sublato scilicet gradu secundo aa. posito $f = \frac{bd+cd-bc}{d-b-c}$, & factis (vt in superiore) mutatione f. in $\frac{bd+cd-bc}{d-b-c}$, & reductione ad communem diuisorem $d-b-c$. & reiectione redundantium ex contradictione, ad exemplum reductionis in 9. Problemate.

PROBLEMA 18.

Æquationē quadrinomiā...

$$\begin{array}{l}aaaa-baaa+bcaa\\-caaa-bdaa\\+daaa-cdaa+bcda\\+faaa-bfaa+bcfa\\-cfaa-bdfa\\+dfaa-cdfa=-bcdf.\end{array}$$

ad trinomiam

$$aaaa+\frac{\begin{array}{l}+bbcaaa\\+bccaaa\\+bddaaa\\+cddaaa\\-bbdaaa\\-ccdaaa\\-2.bcdaaa\end{array}}{bd+cd-bc}\;\frac{\begin{array}{l}-bbccaa\\-bbddaa\\-bcddaa\\-ccddaa\\+bbcdaa\\+bccdaa\end{array}}{bd+cd-bc}=\frac{-bbccdd}{bd+cd-bc}.$$

reducere, sublato scilicet gradu primo a.

Si sit $bcd+bcf = bdf+cdf$. hoc est $bcd = bdf+cdf-bcf$. erit $\frac{bcd}{bd+cd-bc} = f$.

Ponatur

Ponatur igitur $\frac{bcd}{bd+cd-bc} = f$.

Et in æquatione quadrinomia quæ reducenda proponitur, & in particularibus quibus f. inest, fiat primò mutatio f. in $\frac{bcd}{bd+cd-bc}$

Secundò reductio particularium reliquorum ad communem diuiſorem $bd+cd-bc$.

Tertiò reiectio renundantium ex contradictione.

His peractis (vt in 9. Probl.) fit

$$aaaa \begin{array}{l} +bbcaaa \\ +bccaaa \\ +bddaaa \\ +cddaaa \\ -bbdaaa \\ -ccdaaa \\ -2bcdaaa \\ \hline bd+cd-bc \end{array} \begin{array}{l} \\ -bbccaa \\ -bbddaa \\ -bcddaa \\ -ccddaa \\ +bbcdaa \\ +bccdaa \\ \hline bd+cd-bc \end{array} = \frac{-bbccdd}{bd+cd-bc}$$

quæ eſt æquatio trimonia requiſita in qua tollitur gradus primus a.

Et ſic perficitur reductio imperata.

PROBLEMA. 19.

Æquationē quadrinomiā ...

$$\begin{array}{llll} aaaa & -baaa & +bcaa & \\ & -caaa & -bdaa & \\ & +daaa & -cdaa & +bcda \\ & +faaa & -bfaa & +bcfa \\ & & -cfaa & -bdfa \\ & & +dfaa & -cdfa = -bcdf. \end{array}$$

poſito $b+c = d+f$.

ad binomiam

$$\begin{array}{ll} aaaa & -bbba \\ & -bbca \\ & -bcca \\ & -ccca = -bbbc \\ & \qquad\quad -bbcc \\ & \qquad\quad -bccc. \end{array}$$

reducere, ſublatis ſcilicet gradibus aa. & aaa.

PROBLEMA 20.

Æquationē quadrinomiā ...

$$\begin{array}{llll} aaaa & -baaa & +bcaa & \\ & -caaa & -bdaa & \\ & +daaa & -cdaa & +bcda \\ & +faaa & -bfaa & +bcfa \\ & & -cfaa & -bdfa \\ & & +dfaa & -cdfa = -bcdf \end{array}$$

poſito $bc+df = bd+cd+bf+cf$.

PROBLEMA 17.

Reducibilis est quoque quadrinomia superiùs proposita ad trinomiam istam

$$\frac{\begin{matrix} aaaa+bbaaa-bbcca \\ +bcaaa-bcdda \\ +ccaaa-bbdda \\ +ddaaa-ccdda \\ -bdaaa+bbcda \\ -cdaaa+bccda \end{matrix}}{d-b-c\quad d-b-c} = \frac{\begin{matrix} -bbcdd \\ -bccdd \\ +bbccd \end{matrix}}{d-b-c}$$

sublato scilicet gradu secundo aa. posito $f = \frac{bd+cd-bc}{d-b-c}$, & factis (vt in superiore) mutatione f. in $\frac{bd+cd-bc}{d-b-c}$, & reductione ad communem diuisorem $d-b-c$. & reiectione redundantium ex contradictione, ad exemplum reductionis in 9. Problemate.

PROBLEMA 18.

Æquationē quadrinomiā...

$$\begin{matrix} aaaa-baaa+bcaa \\ -caaa-bdaa \\ +daaa-cdaa+bcda \\ +faaa-bfaa+bcfa \\ -cfaa-bdfa \\ +dfaa-cdfa = -bcdf. \end{matrix}$$

ad trinomiam

$$\begin{matrix} aaaa+bbcaaa \\ +bccaaa-bbccaa \\ +bddaaa-bbddaa \\ +cddaaa-bcddaa \\ -bbdaaa-ccddaa \\ -ccdaaa+bbcdaa \\ -2.bcdaaa+bccdaa = -bbccdd \\ \overline{bd+cd-bc}\quad \overline{bd+cd-bc}\quad \overline{bd+cd-bc}. \end{matrix}$$

reducere, sublato scilicet gradu primo a.

Si sit $bcd+bcf = bdf+cdf$, hoc est $bcd = bdf+cdf-bcf$. erit $\frac{bcd}{bd+cd-bc} = f$.

Ponatur

Ponatur igitur $\frac{bcd}{bd+cd-bc} = f$.

Et in æquatione quadrinomia quæ reducenda proponitur, & in particularibus quibus f. inest, fiat primò mutatio f. in $\frac{bcd}{bd+cd-bc}$

Secundò reductio particularium reliquorum ad communem diuisorem $bd+cd-bc$.

Tertiò reiectio renundantium ex contradictione.

His peractis (vt in 9. Probl) fit

$$\begin{array}{l} aaaa+bbcaaa \\ +bccaaa-bbccaa \\ +bddaaa-bbddaa \\ +cddaaa-bcddaa \\ -bbdaaa-ccddaa \\ -ccdaaa+bbcdaa \\ -2\,bcdaaa+bccdaa \end{array} = -\frac{bbccdd}{bd+cd-bc}$$

$$\overline{bd+cd-bc} \quad \overline{bd+cd-bc}$$

quæ est æquatio trimonia requisita in qua tollitur gradus primus a.

Et sic perficitur reductio imperata.

PROBLEMA. 19.

Æquationē quadrinomiā ...

$$\begin{array}{l} aaaa-baaa+bcaa \\ \quad -caaa-bdaa \\ \quad +daaa-cdaa+bcda \\ \quad +faaa-bfaa+bcfa \\ \qquad\quad -cfaa-bdfa \\ \qquad\quad +dfaa-cdfa = -bcdf. \end{array}$$

posito $b+c = d+f$.

ad binomiam

$$\begin{array}{l} aaaa-bbba \\ \quad -bbca \\ \quad -bcca \\ \quad -ccca = -bbbc \\ \qquad\qquad -bbcc \\ \qquad\qquad -bccc. \end{array}$$

reducere, sublatis scilicet gradibus aa. & aaa.

PROBLEMA 20.

Æquationē quadrinomiā ...

$$\begin{array}{l} aaaa-baaa+bcaa \\ \quad -caaa-bdaa \\ \quad +daaa-cdaa+bcda \\ \quad +faaa-bfaa+bcfa \\ \qquad\quad -cfaa-bdfa \\ \qquad\quad +dfaa-cdfa = -bcdf \end{array}$$

posito $bc+df = bd+cd+bf+cf$.

ad binomiam $\frac{aaaa - bbbaaa - bbcaaa - bccaaa - cccaaa}{bb+bc+cc} = \frac{-bbbccc}{bb+bc+cc}$ reducere, sublatis scilicet gradibus a. & aa.

PROBLEMA. 21.

Æquationē quadrinomiā $aaaa - baaa + bcaa - caaa - bdaa + daaa - cdaa + bcda + faaa - bfaa + bcfa - cfaa - bdfa + dfaa - cdfa = - bcdf$.

posito $d + f = b + c$
ad binomiam $aaaa - bbaa - ccaa = - bbcc$. reducere, sublatis scilicet gradibus a. aaa.

Nota.

Tres antecedentes binomiæ reductitiæ Problematum 19 20. 21. licet in ijsdem radicibus explicatorijs b. c. cum trinomijs tribus superioribus problematum 16. 17. 18. ab eadem quadrinomia hic proposita reductis conueniant, vt in propositionibus 32. 33. 34. & 35 36 37. Sectionis quartæ demonstrandum est: reductiones tamen earum cùm in autographis obscuriùs traditæ sint, ad meliorem inquisitionem referendæ sunt.

Corollarium generale.

In reductionibus quæ per Problemata tertiæ huius Sectionis fiunt, nec radicis quæsititiæ a. aut reliquorum graduum, nec elementorum datorum b. c. d. vllam factam esse mutationem, manifestum est.

Æquationum canonicarum reductitiarum quarum reductiones in tertia hac Sectione traduntur recollectio.

Quadratica.

1. . . . . $aa = bb$.

Cubicæ.

1. . . . . $aaa - bba - bca - cca = - bbc - bcc$

2. aaa

2. . . . . $$aaa\,\dfrac{\begin{array}{l} -bbaa \\ -bcaa \\ -ccaa \end{array}}{b+c} = \dfrac{-bbcc}{b+c}$$

3. . . . . $$aaa\,\begin{array}{l} -bba \\ -bca \\ -cca \end{array} = \begin{array}{l} +bbc \\ +bcc \end{array}$$

4. . . . . $$aaa\,\dfrac{\begin{array}{l} +bbaa \\ +bcaa \\ +ccaa \end{array}}{b+c} = \dfrac{+bbcc}{b+c}$$

5. . . . . $aaa + 3.bca = +bbb - ccc$

6. . . . . $aaa + 3.bca = -bbb + ccc$

7. . . . . $aaa - 3.bca = +bbb + ccc$

Biquadraticæ.

1. . . . . $$aaaa\,\begin{array}{ll} -bbaa & +bbca \\ -ccaa & +bbda \\ -ddaa & +bcca \\ -bcaa & +ccda \\ -bdaa & +bdda \\ -cdaa & +cdda \\ & +2\,bcda \end{array} = \begin{array}{l} +bbcd \\ +bccd \\ +bcdd \end{array}$$

2. . . . . $$aaaa\,\begin{array}{ll} -bbaaa & +bbcca \\ -ccaaa & +bbdda \\ -ddaaa & +ccdda \\ -bcaaa & +bcdda \\ -bdaaa & +bccda \\ -cdaaa & +bbcda \\ \hline b+c+d & b+c+d \end{array} = \dfrac{\begin{array}{l} +bbccd \\ +bbcdd \\ +bccdd \end{array}}{b+c+d}$$

3. . . . . $$aaaa\,\begin{array}{ll} -bbcaaa & \\ -bbdaaa & +bbccaa \\ -bccaaa & +bbddaa \\ -bddaaa & +ccddaa \\ -ccdaaa & +bcddaa \\ -cddaaa & +bccdaa \\ -2\,bcdaaa & +bbcdaa \\ \hline bc+bd+cd & bc+bd+cd \end{array} = \dfrac{+bbccdd}{bc+bd+cd}$$

4. aaaa

4. . . . . . .
$$aaaa\begin{array}{l}-bbaa\\-ccaa\\-ddaa\\-bcaa\\-bdaa\\-cdaa\\\quad-2.dcba\end{array}\begin{array}{l}-bbca\\-bbda\\-bcca\\-ccda\\-bdda\\-cdda\\\end{array} = = \begin{array}{l}+bbcd\\+bccd\\+bcdd\end{array}$$

5. . . . . . .
$$aaaa\frac{\begin{array}{l}+bbaaa\\+ccaaa\\+ddaaa\\+bcaaa\\+bdaaa\\+cdaaa\end{array}}{b+c+d}\frac{\begin{array}{l}-bbcca\\-bbada\\-ccdda\\-bcdda\\-bccda\\-bbcda\end{array}}{b+c+d} = = \frac{\begin{array}{l}+bbccd\\+bbcdd\\+bccdd\end{array}}{b+c+d}$$

6. . . . . . .
$$aaaa\frac{\begin{array}{l}+bbcaaa\\+bbdaaa\\+bccaaa\\+ccdaaa\\+bddaaa\\+cddaaa\\+2.bcdaaa\end{array}}{bc+bd+cd}\frac{\begin{array}{l}\\+bbccaa\\+bbddaa\\+ccddaa\\+bcddaa\\+bccdaa\\+bbcdaa\end{array}}{bc+bd+cd} = = \frac{+bbccdd}{bc+bd+cd}$$

7. . . . . . .
$$aaaa\begin{array}{l}-bdaa\\-cdaa\\+bbaa\\+bcaa\\+ccaa\\+ddaa\\\quad+2.bcda\end{array}\begin{array}{l}-bbca\\-bcca\\-bdda\\-cdda\\+bbda\\+ccda\\\end{array} = = \begin{array}{l}+bbcd\\+bccd\\-bcdd\end{array}$$

8. . . . .
$$aaaa\frac{\begin{array}{l}-bbaaa\\-bcaaa\\-ccaaa\\-ddaaa\\+bdaaa\\+cdaaa\end{array}}{b+c-d}\frac{\begin{array}{l}+bbcca\\+bbdda\\+bcdda\\+ccdda\\-bbcda\\-bccda\end{array}}{b+c-d} = = \frac{\begin{array}{l}-bbccd\\+bbcdd\\+bccdd\end{array}}{b+c+a}$$

9\. . . . . . .
$$\begin{array}{lll} aaaa + bbaaa - bbcca & & \\ \quad + bcaaa - bbdda & & \\ \quad + ccaaa - bcdda & & \\ \quad + ddaaa - ccdda & & \\ \quad - bdaaa + bbcda & & \\ \quad \underline{- cdaaa} \; \underline{+ bccda} & = & -bbcdd \\ \quad d - b - c \quad d - b - c & & -bccdd \\ & & \underline{+bbccd} \\ & & d - b - c \end{array}$$

10\. . . . . . .
$$\begin{array}{lll} aaaa + bbcaaa & & \\ \quad + bccaaa - bbcca & & \\ \quad + bddaaa - bbdda & & \\ \quad + cddaaa - bcdda & & \\ \quad - bbdaaa - ccdda & & \\ \quad - ccdaaa + bbcda & & \\ \quad \underline{- 2.\,bcdaaa} \; \underline{+ bccda} & = & \underline{-bbccdd} \\ \quad bd + cd - bc \quad bd + cd - bc & & bd + cd - bc \end{array}$$

11\. . . . . . .
$$\begin{array}{lll} aaaa - bbba & & \\ \quad - bbca & & \\ \quad - bcca & & \\ \quad - ccca & = & -bbbc \\ & & -bbcc \\ & & -bccc \end{array}$$

12\. . . . . . .
$$\begin{array}{lll} aaaa - bbbaaa & & \\ \quad - bbcaaa & & \\ \quad - bccaaa & & \\ \quad \underline{- cccaaa} & = & \underline{-bbbcc} \\ \quad bb + bc + cc & & bb + bc + cc \end{array}$$

13\. . . . . . .
$$\begin{array}{lll} aaaa - bbaa & & \\ \quad - ccaa & = & -bbcc \end{array}$$

Collectio æquationum aliquarum canonicarum cum tali dispositione vt de facili appareat generatio aliorum sublimiorum graduum.

$$\begin{array}{lll} +bc & = & +ba \\ & & +ca - aa \end{array}$$

$$\begin{array}{lll} +bbc & = & +bba \\ +bcc & & +bca \\ & & +cca - aaa \end{array}$$

$$\begin{array}{lll} +bbbc & = & +bbba \\ +bbcc & & +bbca \\ +bccc & & +bcca \\ & & +ccca - aaaa \end{array}$$

$$\begin{array}{lcl} +bbbbc & = & +bbbba \\ +bbbcc & & +bbbca \\ +bbccc & & +bbcca \\ +bcccc & & +bccca \\ & & +cccca - aaaaa \end{array}$$

Et eâdem in infinitum methodo.

$$\begin{array}{lcl} +bbcc & = & +bbaa \\ \overline{b+c} & & +bcaa \\ & & +ccaa - aaa \\ & & \overline{b+c} \end{array}$$

$$\begin{array}{lcl} +bbbcc & = & +bbbaa \\ +bbccc & & +bbcaa \\ \overline{b+c} & & +bccaa \\ & & +cccaa - aaaa. \\ & & \overline{b+c} \end{array}$$

$$\begin{array}{lcl} +bbbbcc & = & +bbbbaa \\ +bbbccc & & +bbbcaa \\ +bbcccc & & +bbccaa \\ \overline{b+c} & & +bcccaa \\ & & +ccccaa - aaaaa. \\ & & \overline{b+c} \end{array}$$

Et eâdem in infinitum methodo.

$$\begin{array}{lcl} +bbbccc & = & +bbbaaa \\ \overline{bb+bc+cc} & & +bbcaaa \\ & & +bccaaa \\ & & +cccaaa - aaaaa. \\ & & \overline{bb+bc+cc} \end{array}$$

$$\begin{array}{lcl} +bbbbccc & = & +bbbbaaa \\ +bbbcccc & & +bbbcaaa \\ \overline{bb+bc+cc} & & +bbccaaa \\ & & +bcccaaa \\ & & +ccccaaa - aaaaa. \\ & & \overline{bb+bc+cc} \end{array}$$

Et sic de cæteris eâdem in infinitum methodo.

$$\begin{array}{lcl} +bbbbccc & = & +bbbbaaaa \\ \overline{bbb+bbc+bcc+ccc} & & +bbbcaaaa \\ & & +bbccaaaa \\ & & +bcccaaaa \\ & & +ccccaaaa - aaaaa. \\ & & \overline{bbb+bbc+bcc+ccc} \end{array}$$

Et sic de cæteris eâdem in infinitum methodo.

Alia

Alia collectio & series Canonicarum.

$$\begin{array}{lcl}
+bcd & = & +bca-baa \\
 & & +bda-caa \\
 & & +cda-daa+aaa
\end{array}$$

$$\begin{array}{lcl}
+bbcd & = & +bbca \\
+cbcd & & +bbda-bbaa \\
+dbcd & & +ccba-ccaa \\
 & & +ccda-ddaa \\
 & & +ddba-bcaa \\
 & & +ddca-bdaa \\
 & & +2bcda-cdaa+aaaa.
\end{array}$$

$$\begin{array}{lcl}
+bcbcd & = & +bbcca-bbaaa \\
+bdbcd & & +bbdda-ccaaa \\
+cdbcd & & +ccdda-ddaaa \\
\overline{b+c+d} & & +bbcda-bcaaa \\
 & & +cbcda-bdaaa \\
 & & +dbcda-cdaaa+aaaa. \\
 & & \overline{b+c+d}\quad\overline{b+c+d}
\end{array}$$

$$\begin{array}{lcl}
+bcdbcd & = & +bbccaa-bbcaaa \\
 & & +bbddaa-bbdaaa \\
 & & +ccddaa-ccbaaa \\
 & & +bbcdaa-ccdaaa \\
 & & +cbcdaa-ddbaaa \\
 & & +dbcdaa-ddcaaa \\
 & & bc+bd+cd-2.bcdaaa+aaaa \\
 & & \qquad\qquad\overline{bc+bd+cd}
\end{array}$$

$$\begin{array}{lcl}
+bbcbcd & = & +bbbcca \\
+bbdbcd & & +bbbdda \\
+ccbbcd & & +cccbba-bbbaaa \\
+ccdbcd & & +bbbcda-cccaaa \\
+ddbbcd & & +cccdda-dddaaa \\
+ddcbcd & & +cccbda-bbcaaa \\
+2.bcdbcd & & +dddbba-bbdaaa \\
\overline{b+c+d} & & +dddcca-ccbaaa \\
 & & +dddbca-ccdaaa \\
 & & +2.bcbcba-ddbaaa \\
 & & +2.bdbcba-ddcaaa \\
 & & +2.bdbcda-bcdaaa+aaaaa. \\
 & & \overline{b+c+d}\quad\overline{b+c+d}
\end{array}$$

Æquationum

SECTIO QVARTA.

Æquationum canonicarum tam primariarum quam *secundariarum, radicum designatio.*

PROPOSTIO 1.

Æquationis $\begin{array}{l} aa - ba \\ \qquad + ca = + bc. \end{array}$ est b. radix radici quæsititiæ a. æqualis.

Nam si æquationis $\begin{array}{l} aa - ba \\ \qquad + ca = + bb. \end{array}$ radici a. ponatur b. æqualis, mutata a. in b. erit $\begin{array}{l} bb - bb \\ \qquad + cb = + cb. \end{array}$

Est autem æqualitas ista per se manifesta.

Ergo radici a. posita b. æqualis, æqualis est. vt est enunciatum.

Quod autem non detur radix alia præter b. æquationis radici a. æqualis in sequenti Lemmate demonstratur.

Lemma.

Si dari possit radix aliqua æquationis radici a. æqualis quæ radici b. inæqualis sit, esto illa c. siue alia quæcunque.

Posito igitur $c = a$. erit . . . $\begin{array}{l} cc - bc \\ \qquad + cc = + bc. \end{array}$

Ergo . . . . $cc + cc = + bc + bc.$

Ergo . . . . $\dfrac{c + c}{c} = \dfrac{c + c}{b}$

Ergo . . . . $c = b$. Quod est contra hypothesim.

Non est igitur $c = a$. vt erat positum. Quod de alia quacunque præter b. similiter demonstrari potest.

PROPOSITIO 2.

Æquationis $\begin{array}{l} aa - ba \\ \qquad - ca = - bc. \end{array}$ sunt b. vel c. radices, radici quæsitiæ a. æquales.

Nam si æquationis $\begin{array}{l} aa. - ba \\ \qquad - ca = - bc. \end{array}$ radici a. ponatur b. æqualis, mutata a. in b. erit $\begin{array}{l} bb - bb \\ \qquad - cb = - bc \end{array}$

Est

Est autem æqualitas ista per se manifesta.

Ergo radici *a.* posita *b.* æqualis, æqualis est.

Item si radici *a.* ponatur *c.* æqualis, mutata *a.* in *c.* erit $\begin{array}{l} cc-bc \\ -cc = -bc. \end{array}$

Est item æqualitas hæc per se manifesta.

Ergo radici *a.* posita *c.* æqualis, æqualis quoque est.

Sunt igitur *b.* vel *c.* radici quæsititiæ *a.* æquales, vt est enunciatum.

Quod autem non detur radix alia præter *b.* vel *c.* radici *a.* æqualis in sequenti Lemmate demonstratur.

Lemma.

Si dari possit radix aliqua radici *a.* æqualis, quæ radicibus *b.* vel *c.* inæqualis sit, esto illa *d.* siue alia quæcunque.

Posito igitur $d = a.$ erit $\begin{array}{l} dd-bd \\ -cd = -bc. \end{array}$

Ergo . . . . $dd-cd = +bd-bc.$

Ergo . . . . $\begin{array}{r|} +d-c \\ d \\ \hline \end{array} = \begin{array}{r|} +d-c \\ b \\ \hline \end{array}$

Ergo . . . . $d = b.$ Quod est contra hypothesim.

Vel erit . . . . $+dd-bd = +cd-bc$

Ergo . . . . . $\begin{array}{r|} -d-b \\ d \\ \hline \end{array} = \begin{array}{r|} -d-b \\ c \\ \hline \end{array}$

Ergo . . . . $d = c.$ Quod est etiam contra hypothesim.

Non est igitur $d = a.$ vt erat positum. Quod de alia quacunq; præter *b.* vel *c.* similiter demonstrari potest.

PROPOSITIO 3.

Æquationis $\begin{array}{l} aaa+baa+bca \\ \quad +caa-bda \\ \quad -daa-cda = +bcd. \end{array}$ est *d.* radix, radici quæsititiæ *a.* æqualis.

Nam si æquationis $\begin{array}{l} aaa+baa+bca \\ \quad +caa-bda \\ \quad -daa-cda = +bcd. \end{array}$ radici *a* ponatur *d.* æqualis, mutata *a.* in *d.* erit $\begin{array}{l} ddd+bdd+bcd \\ \quad +cdd-bdd \\ \quad -ddd-cdd = +bcd. \end{array}$

Est autem æqualitas ista reiectis contradictorijs manifesta.

Ergo radici $a.$ posita $d.$ æqualis, æqualis est, vt est enunciatum.

Quod autem non detur radix alia præter $d.$ æquationis radici $a.$ æqualis in sequenti Lemmate demonstratur.

Lemma.

Si dari possit radix aliqua radici $a.$ æqualis, quæ radici $d.$ inæqualis sit, esto illa $b.$ vel $c.$ vel alia quæcunque.

Posito igitur $c = a.$ erit

$$\begin{array}{l} ccc + bcc + bcc \\ + ccc - bdc \\ - dcc - cdc = + bcd. \end{array}$$

Ergo ordinatis particularibus $+ 2.ccc + 2.bcc = + 2.ccd + 2.bcd.$

Ergo . . . . $\dfrac{+cc+bc}{c} = \dfrac{+cc+bc}{d}$

Ergo . . . . $c = d.$ quod est contra hypothesim.

Non est igitur $c = a.$ vt erat positum. Quod etiam de $b.$ vel quacunque alia præter $d.$ consimili ratiocinio concludendum est.

PROPOSITIO 4.

Æquationis

$$\begin{array}{l} aaa + baa - bca \\ - caa - bda \\ - daa + cda = - bcd. \end{array}$$

sunt $c.$ vel $d.$ radices explicatoriæ radici quæsititiæ $a.$ æquales.

Nam si æquationis

$$\begin{array}{l} aaa + baa - bca \\ - caa - bda \\ - daa + cda = - bcd. \end{array}$$

radici $a.$ ponatur $c.$ æqualis, mutata $a.$ in $c.$ erit

$$\begin{array}{l} ccc + bcc - bcc \\ - ccc - bdc \\ - dcc + cdc = - bcd. \end{array}$$

Est autem æqualitatis huius veritas separatis redundantibus manifesta.

Ergo radici $a.$ posita $c.$ æqualis, æqualis est.

Item si radici $a.$ ponatur $d.$ æqualis, mutata $a.$ in $d.$ erit

$$\begin{array}{l} ddd + bdd - bcd \\ - cdd - bdd \\ - ddd + cdd = - bcd. \end{array}$$

Est autem æqualitatis huius veritas similiter manifesta.

Ergo radici $a.$ posita $d.$ æqualis, æqualis quoque est.

Sunt igitur radices $c.$ & $d.$ radici quæsititiæ $a.$ æquales, vt est enunciatum.

Quod

Quod autem non detur radix alia præter $c.$ & $d.$ æquationis radici $a.$ æqualis in sequenti Lemmate demonstratur.

Lemma.

Si dari possit radix aliqua radici $a.$ æqualis, quæ radicibus $c.$ vel $d.$ inæqualis sit, esto illa $b.$ vel alia quæcunque.

Posito igitur $b = a.$ erit
$$\begin{array}{l} bbb+bbb-bcb \\ -cbb-bdb \\ -dbb+cdb = -bcd. \end{array}$$

Ergo ordinatis particularibus $+2.bbb-2.bbd = +2.cbb-2.cbd.$

Hoc est . . . . $+bbb-bbd = +cbb-cbd.$

Ergo . . . . $\frac{+bb-bd}{b} = \frac{+bb-bd}{c}$

Ergo . . . . $b = c.$ Quod est contra hypothesim.

Vel erit . . . . $+2.bbb-2.bbc = +2.dbb-2.dbc.$

Hoc est . . . . $+bbb-bbc = +2.dbb-dbc.$

Ergo . . . . $\frac{+bb-bc}{b} = \frac{+bb-bc}{d}$

Ergo . . . $b = d.$ quod est quoque contra hypothesim.

Non est igitur $b = a.$ vt erat positum. Quod de quacunq; alia præter $c.$ & $d.$ consimili deductione demonstrari potest.

Consectarium.

Binas æquationes in duobus antecedentibus theorematis propositas coniugatas esse ipso intuitu patet.

Sunt enim . . .
$$\begin{array}{l} +aaa-baa-bca \\ +caa-bda \\ +daa+cda = +bcd = +bca-baa \\ \qquad\qquad\qquad\qquad +bda+baa \\ \qquad\qquad\qquad\qquad -cda+daa-aaa. \end{array}$$

Radicum autem suarum habitudo ex theorematis innotescit, primæ scilicet $a = b.$ Secundæ vero $a = c.$ vel $d.$ quod adnotandum erat.

PROPOSITIO 5.

Æquationis
$$\begin{array}{l} aaa-baa+bca \\ -caa+bda \\ -daa+cda = +bcd. \end{array}$$
sunt $b.$ vel $c.$ vel $d.$ radices explicatoriæ, radici quæsititiæ $a.$ æquales.

Nam

Nam si æquationis $aaa - baa + bca - caa + bda - daa + cda = + bcd$. radici a. ponatur b. æqualis, mutata a. in b. erit $bbb - bbb + bcb - cbb + bdb - dbb + cdb$

Est autem æqualitas ista abstractis contradictorijs manifesta.

Ergo radici a. posita b. æqualis, æqualis est.

Item si radici a. ponatur c. æqualis, mutata a. in c. erit

$$ccc - bcc + bcc - ccc + bdc - dcc + cdc = + bcd.$$

Est autem æqualitas hæc reiectis contradictorijs manifesta.

Ergo radici a. posita c. æqualis, æqualis est.

Item si radici a. ponatur d. æqualis, mutata a. in d. erit

$$ddd - bdd + bcd - cdd + bdd - ddd + cdd = + bcd.$$

Est autem æqualitas ista reiectis contradictorijs manifesta.

Ergo radici a. posita d. æqualis, æqualis quoque est.

Sunt igitur radices b. vel c. vel d. radici quæsititiæ a. æquales, vt est enunciatum.

Quod autem non detur radix alia præter b. vel c. vel d. radici a, æqualis in sequenti Lemmate demonstratur.

Lemma.

Si dari possit radix alqua radici a. æqualis, quæ radicibus b. vel c. vel d. inæqualis sit, esto illa f. vel alia quæcunque.

Posito igitur $f = a$. erit $fff - bff + bcf - cff + bdf - dff + cdf = + bcd$.

Ergo ordinatis particularibus est $fff - cff + cdf - dff = + bff - bcf + bcd - bdf$.

Ergo . . . $\dfrac{ff - cf + cd - df}{f} = \dfrac{+ ff - cf + cd - df}{b}$

Ergo . . . . $f = b$. Quod est contra hypothesim.

Vel mutata ordinatione est $fff - bff + bdf - dff = cff - cbf + cbd - cdf$.

Ergo . . . . $\dfrac{ff - bf + bd - df}{f} = \dfrac{ff - bf + bd - df}{c}$

Ergo . . . $f = c$. Quod est etiam hypothesim.

Vel mutata adhuc ordinatione est $fff - bff + bcf - cff = dff - dbf + dbc - dcf$.

Ergo

Ergo . . . $\dfrac{ff-bf+bc-cf}{f} = \dfrac{ff-bf+bc-cf}{d}$

Ergo . . . . $f = d$. Quod est etiam contra hypothesin.

Non est igitur $f = a$. vt erat positum. Quod de alia quacunque præter $b.\ c.\ d.$ simili ratiocinio concludere licet.

Reductitiæ.

PROPOSITIO 6.

Æquationis
$$\begin{array}{l} aaa-bba \\ \quad -bca \\ \quad -cca = -bbc \\ \qquad\qquad\quad -bcc. \end{array}$$
sunt b. vel c. radices radici quæsititiæ a. æquales.

Nam si ponatur $b = a$. & in æquatione proposita, mutetur a. in b. erit
$$\begin{array}{l} bbb-bbb \\ \quad -bbc \\ \quad -bcc = -bbc \\ \qquad\qquad\quad -bcc \end{array}$$

Vel si ponatur $c = a$. & mutetur a. in c. erit
$$\begin{array}{l} ccc-bbc \\ \quad -bcc \\ \quad -ccc = -bbc \\ \qquad\qquad\quad -bcc \end{array}$$

Æqualitates autem istæ reiectis contradicentibus manifestæ sunt.

Est igitur æquationis propositæ radix quæsititia $a = b$. vel c. vt est enunciatum.

PROPOSITIO 7.

Æquationis . . .
$$\begin{array}{l} aaa-bba \\ \quad -bca \\ \quad -cca = +bbc \\ \qquad\qquad\quad +bcc. \end{array}$$
est $b+c$. radix radici quæsititiæ a. æqualis.

Nam si ponatur $b+c = a$. & in æquatione proposita mutetur a. in $b+c$.

erit

$$\text{erit} \ldots \begin{array}{l} +bbb-bbb \\ +3.bbc-bbc \\ +3.bcc-bbc \\ +ccc-bcc \\ -bcc \\ -ccc = \begin{array}{l}+bbc. \\ +bcc\end{array} \end{array}$$

Reiectis autem contradicentibus æqualitas manifesta est, scil. $\begin{array}{l}+bbc \\ +bcc\end{array} = \begin{array}{l}+bbc \\ +bcc.\end{array}$

Ergo radici $a.$ posita $b+c$ æqualis, æqualis est. vt est enunciatum.

Consectarium.

Hinc patet æquationem istam alteri proximè antecedenti coniugatam esse.

$$\text{Sunt enim} \ldots \begin{array}{l} aaa-bba \\ -bca \\ -cca = \begin{array}{l}+bbc \\ +bcc = \begin{array}{l}+bba \\ +bca \\ +cca-aaa.\end{array}\end{array}\end{array}$$

Et in prima $a = b+c.$ in secunda $a = b.$ vel $c.$ quod adnotasse sufficiat.

PROPOSITIO 8.

$$\text{Æquationis} \ldots \frac{\begin{array}{l} aaa-bbaa \\ -bcaa \\ -ccaa \end{array}}{b+c.} = \frac{-bbcc.}{b+c} \quad \text{est } b. \text{ vel } c.$$

radix, radici quæsititiæ $a.$ æqualis.

$$\text{Nam si ponatur } b = a. \text{ erit} \ldots \frac{\begin{array}{l}+bbbb \\ +cbbb\end{array}}{b+c} \begin{array}{l}-bbbb \\ -cbbb \\ -ccbb\end{array} \Big/ b+c = \frac{-bbcc}{b+c}$$

$$\text{Vel posita } c = a \text{ erit} \ldots \frac{\begin{array}{l}+bccc \\ +cccc\end{array}}{b+c} \begin{array}{l}-bbcc \\ -bccc \\ -cccc\end{array} \Big/ b+c = \frac{-bbcc}{b+c}$$

Æqualitates autem istæ manifestæ sunt.

Est igitur propositæ æquationis radix $a = b.$ vel $c.$ vt est enunciatum.

PRO-

PROPOSITIO 9.

Æquationis $aaa + \dfrac{bbaa + bcaa + ccaa}{b+c} = \dfrac{+bbcc}{b+c}$ est $\dfrac{bc}{b+c}$ radix, radici quæsititiæ a. æqualis.

Nam (per Probl. 5. Sectionis 3.) æquatio binomia hic proposita à trinomia sua reducitur, posito $\dfrac{bc}{b+c} = d$. & mutatis altera in alteram.

Sed (per Prop. 3. huius) est trinomiæ illius radix $a = d$.

Est igitur æquationis huius binomiæ radix $a = \dfrac{bc}{b+c}$ vt est enunciatum.

Consectarium.

Hinc patet æquationem istam æquationi proximè antecedenti coniugatam esse.

Sunt enim $aaa + \dfrac{bbaa + bcaa + ccaa}{b+c} = \dfrac{+bbcc}{b+c} = \dfrac{+bbaa + bcaa + ccaa - aaaa}{b+c}$

Et in prima $a = \dfrac{bc}{b+c}$ in secunda $a = b$. vel c. quod adnotasse sufficiat.

PROPOSITIO 10.

Æquationis $aaa + 3.baa + 3.bba = +ccc - bbb$. est radix $c - b$. radici quæsititiæ a. æqualis.

Nam si æquationis $aaa + 3.baa + 3.bba = +ccc - bbb$ radici a. ponatur $c - b$ æqualis mutata a. in $c - b$. erit

$$\left.\begin{array}{ll} +ccc - 3.bcc + 3.bbc - bbb = aaa \\ \text{Et} \quad +3.bcc - 6.bbc + 3.bbb = +3.baa \\ \text{Et} \quad +3.bbc - 3.bbb = +3.bba \end{array}\right\} = +ccc - bbb.$$

Æqualitas autem ista reiectis contradictorijs manifesta est.

Ergo radici a. posita $c - b$. æqualis, æqualis est. vt est enunciatum.

PRO-

PROPOSITIO 11.

Æquationis $aaa - 3.baa + 3.bba = +ccc + bbb.$ est radix $c + b.$ radici quæsititiæ $a.$ æqualis.

Nam si æquationis $aaa - 3.baa + 3.bba = +ccc + bbb.$ radici $a.$ ponatur $c + b.$ æqualis, mutata $a.$ in $c + b.$ erit

$$\left.\begin{array}{lll} +ccc + 3.bcc + 3.bbc + bbb & = & +aaa \\ \text{Et} \;.\; . - 3.bcc - 6.bbc - 3.bbb & = & -3.baa \\ \text{Et} \;.\;.\;.\;.\;. + 3.bbc + 3.bbb & = & +3.bba \end{array}\right\} = +ccc + bbb$$

Æqualitas autem ista reiectis contradictorijs manifesta est.

Ergo radici $a.$ posita $c + b.$ æqualis, æqualis est. vt est enunciatum.

PROPOSITIO 12.

Æquationis $aaa - 3.baa + 3.bba = +bbb - ccc.$ est radix $b - c.$ radici quæsititiæ $a.$ æqualis.

Nam si æquationis $aaa - 3.baa + 3.bba = +bbb - ccc.$ radici $a.$ ponatur $b - c.$ æqualis, mutata $a.$ in $c + b.$ erit

$$\left.\begin{array}{lll} -ccc + 3.bcc - 3bbc + bbb & = & +aaa \\ \text{Et} \;.\; . - 3.bcc + 6.bbc - 3.bbb & = & -3.baa \\ \text{Et} \;.\;.\;.\;.\;.\; - 3bbc + 3.bbb & = & +3bba \end{array}\right\} = +bbb - ccc.$$

Æqualitas autem ista reiectis contradictorijs manifesta est.

Ergo radici $a.$ posita $b - c.$ æqualis, æqualis est. Vt est enunciatum.

PROPOSITIO 13.

Æquationis $aaa - 3.baa + 3.bba = +2.bbb$ est $2.b.$ radix radici quæsititiæ $a.$ æqualis.

Nam si præpositæ æquationis radici $a.$ ponatur $2.b.$ æqualis, mutata $a.$ in $2.b.$ erit $+8\,bbb - 12.bbb + 6.bbb = +2.bbb.$

Æqualitas autem ista per se manifesta est.

Ergo radici $a.$ posita $2.b.$ æqualis, æqualis est. Vt est enunciatum.

Reductitiæ.

PROPOSITIO 14.

Æquationis $aaa + 3.bca = +ccc - bbb$. est radix $c - b$. radici quæsititiæ a. æqualis.

Nam si æquationis $aaa + 3.bca = +ccc - bbb$. radici a. ponatur $c - b$. æqualis, mutata a. in $c - b$.

$$\left.\begin{array}{ll} \text{Erit} \;.\;. & ccc - 3.bcc + 3.bbc - bbb = +aaa \\ \text{Et} \;.\;.\;.\;.\;. & +3.bcc - 3.bbc = +3.bca \end{array}\right\} = +ccc - bbb$$

Æqualitas autem ista reiectis contradictorijs manifesta est.

Ergo radici a. posita $c - b$. æqualis, æqualis est, vt est enunciatum.

PROPOSITIO 15.

Æquationis $aaa - 3.bca = +ccc + bbb$. est radix $c + b$. radici quæsititiæ a. æqualis.

Nam si æquationis $aaa - 3.bca = +ccc + bbb$. radici a. ponatur $c + b$. æqualis, mutata a. in $c + b$.

$$\left.\begin{array}{ll} \text{Erit} & ccc + 3.bcc + 3.bbc + bbb = +aaa \\ \text{Et} \;.\;.\;. & -3.bcc - 3.bbc = -3.bca \end{array}\right\} = +ccc + bbb.$$

Æqualitas autem ista reiectis contradictorijs manifesta est.

Ergo radici a. posita $c + b$. æqualis, æqualis est, vt est enunciatum.

PROPOSITIO 16.

Æquationis $aaa + 3bca = -ccc + bbb$. est radix $b - c$. radici quæsititiæ a. æqualis.

Nam si æquationis $aaa + 3.bca = -ccc + bbb$. radici a. ponatur $b - c$. æqualis, mutata a. in $b - c$.

$$\left.\begin{array}{ll} \text{Erit} & bbb - 3.cbb + 3.ccb - ccc = +aaa \\ \text{Et} \;.\;.\;. & +3.cbb - 3.ccb = +3.bca \end{array}\right\} = -ccc + bbb.$$

Æqualitas autem ista reiectis contradictorijs manifesta est.

Ergo radici a. posita $b - c$. æqualis, æqualis est, vt est enunciatum.

PROPOSITIO 17.

Æquationis $aaa - 3.bba = +2.bbb.$ est radix $2.b.$ radici quæsititiæ $a.$ æqualis.

Nam si æquationis $aaa - 3.bba = +2.bbb.$ radici $a.$ ponatur $2.b.$ æqualis, mutata a in $2b.$ erit $8.bbb - 6.bbb = +2.bbb.$

Est autem æqualitas ista per se manifesta.

Ergo radici $a.$ posita $2.b.$ æqualis, æqualis est, vt est enunciatum.

Reciprocæ.

PROPOSITIO 18.

Æquationis $aaa - baa + cda = +bcd.$ est radix $b.$ radici quæsititiæ $a.$ æqualis.

Nam si æquationis $aaa - baa + cda = +bcd.$ radici $a.$ ponatur $b.$ æqualis, mutata $a.$ in $b.$ erit $bbb - bbb + cdb = +bcd.$

Æqualitas autem ista per se manifesta est.

Ergo radici $a.$ posita $b.$ æqualis, æqualis est, vt est enunciatum.

PROPOSITIO 19.

Æquationis $aaa + baa - cca = +bcc.$ est $c.$ radix radici quæsititiæ $a.$ æqualis.

Nam si æquationis $aaa + baa - cca = +bcc.$ radici $a.$ ponatur $c.$ æqualis, mutata $a.$ in $c.$ erit $ccc + bcc - ccc = +bcc.$

Est autem æqualitas ista per se manifesta.

Ergo radici $a.$ posita $c.$ æqualis, æqualis est, vt est enunciatum.

PROPOSITIO 20.

Æquationis $aaa - baa - cca = -bcc.$ sunt $b.$ vel $c.$ radices radici quæsititiæ $a.$ æquales.

Nam si æquationis $aaa - baa - cca = -bcc.$ radici $a.$ ponatur $b.$ æqualis, mutata $a.$ in $b.$ erit $bbb - bbb - ccb = -bcc.$

Est

Est autem æqualitas ista per se manifesta.

Ergo radici $a.$ posita $b.$ æqualis, æqualis est.

Item si radici $a.$ ponatur $c.$ æqualis, mutata $a.$ in $c.$ erit $ccc - bcc - ccc = - bcc.$

Est etiam æqualitas hæc per se manifesta.

Ergo radici $a.$ posita $c.$ æqualis, æqualis quoque est.

Sunt igitur $b.$ vel $c.$ radices radici quæsititiæ $a.$ æquales, vt est enunciatum.

PROPOSITIO 21.

Æquationis
$$\begin{array}{llll}
aaaa & +baaa & +bcaa & \\
 & +caaa & +bdaa & \\
 & +daaa & +cdaa & +bcda \\
 & -faaa & -bfaa & -bcfa \\
 & & -cfaa & -bdfa \\
 & & -dfaa & -cdfa = +bcdf.
\end{array}$$
est $f.$ radix radici quæsititiæ $a.$ æqualis.

Nam si propositæ æquationis radici $a.$ ponatur $f.$ æqualis, mutata $a.$ in $f.$ erit . . .
$$\begin{array}{llll}
ffff & +bfff & +bcff & \\
 & +cfff & +bdff & \\
 & +dfff & +cdff & +bcdf \\
 & -ffff & -bfff & -bcff \\
 & & -cfff & -bdff \\
 & & -dfff & -cdff = +bcdf
\end{array}$$

Æqualitas autem ista reiectis contradictorijs manifesta est.

Ergo radici $a.$ posita $f.$ æqualis, æqualis est, vt est enunciatum.

Quod autem non detur radix alia præter $f.$ æquationis radici $a.$ æqualis in sequenti Lemmate demonstratur.

Lemma.

Si dari possit radix aliqua æquationis radici $a.$ æqualis, quæ radici $f.$ inæqualis sit, esto illa $b.$ vel $c.$ vel $d.$ vel alia quæcunque.

Posito igitur $b = a.$ erit
$$\begin{array}{llll}
bbbb & +bbbb & +bbbc & \\
 & +cbbb & +bbbd & \\
 & +dbbb & +bbcd & +bbcd \\
 & -fbbb & -bbbf & -bbcf \\
 & & -bbcf & -bbdf \\
 & & -bbdf & -bcdf = +bcdf.
\end{array}$$

Ergo . . . .
$$\begin{array}{c}
+2.bbbb+2.bbbc+2.bbbd+2.bbcd \\
\hline
|| \\
\hline
+2.bbbf+2.bbcf+2.bbdf+2.bcdf
\end{array}$$

Hoc

Hoc est . . . . $\dfrac{+bbbb+bbbc+bbbd+bbcd}{+bbbf+bbcf+bbdf+bcdf}$ ||

Ergo . . $\dfrac{+bbb+bbc+bbd+bcd}{b} = \dfrac{+bbb+bbc+bbd+bcd}{f}$

Ergo . . . . $c = f$. Quod est contra Lemmatis hypothesim.

Non est igitur $b = a$. vt erat positum; Quod de c. & d. quoque vel de quacunque alia præter f. ex deductione concludere licet.

PROPOSITIO 22.

Æquationis
$$\begin{array}{llll} aaaa & -baaa & +bcaa & \\ & -caaa & +bdaa & \\ & -daaa & +cdaa & -bcda \\ & +faaa & -bfaa & +bcfa \\ & & -cfaa & +bdfa \\ & & -dfaa & +cdfa = +bcdf. \end{array}$$
est radix b. vel c. vel d. radici quæsititiæ a. æqualis.

Nam si propositæ æquationis radici a. ponatur b. æqualis, mutata a. in b. erit
$$\begin{array}{llll} bbbb & -bbbb & +bcbb & \\ & -cbbb & +bdbb & \\ & -dbbb & +cdbb & -bcdb \\ & +fbbb & -bfbb & +bcfb \\ & & -cfbb & +bdbb \\ & & -dfbb & +cdfb = +bcdf. \end{array}$$

Æqualitas autem ista reiectis contradictorijs manifesta est.

Ergo radici a. posita b. æqualis, æqualis est.

Item si radici a. ponatur c. æqualis, mutata a. in c. erit
$$\begin{array}{llll} cccc & -bccc & +bccc & \\ & -cccc & +bdcc & \\ & -dccc & +cdcc & -bcdc \\ & +fccc & -bfcc & +bcfc \\ & & -cfcc & +bdfc \\ & & -dfcc & +cdfc = +bcdf. \end{array}$$

Æqualitas autem ista reiectis contradictorijs manifesta est.

Ergo radici a. posita c. æqualis, æqualis est.

Item si radici a. ponatur d. æqualis, mutata a. in d. erit
$$\begin{array}{llll} dddd & -bddd & +bcdd & \\ & -cddd & +bddd & \\ & -dddd & +cddd & -bcdd \\ & +fddd & -bfdd & +bcfd \\ & & -cfdd & +bdfd \\ & & -dfdd & +cdfd = +bcdf. \end{array}$$

Æqua-

Æqualitas autem ista reiectis contradictorijs manifesta est.

Ergo radici *a.* posita *d.* æqualis, æqualis est.

Sunt igitur radices *b. c. d.* radici quæsititiæ *a.* æquales, vt est enunciatum.

Quod autem non detur radix alia præter *b. c. d.* æquationis radici *a.* æqualis, in sequenti Lemmate demonstratur.

Lemma.

Si dari possit radix aliqua æquationis radici *a.* æqualis, quæ radicibus *b. c. d.* inæqualis sit, esto illa *f.* siue alia quæcunque.

Posito igitur $f = a.$ erit

$$\begin{array}{l} ffff - bfff + bcff \\ \quad - cfff + bdff \\ \quad - dfff + cdff - bcdf \\ \quad + ffff - bfff + bcff \\ \qquad\quad - cfff + bdff \\ \qquad\quad - dfff + cdff = + bcdf. \end{array}$$

Ergo . . . . . $\dfrac{+2.ffff - 2.cfff + 2.cdff - 2.dfff}{+2.bfff - 2.bcff + 2.bcdf - 2.bdff}$ (||)

Hoc est . . . . . . $\dfrac{+ffff - cfff + cdff - dfff}{+bfff - bcff + bcdf - bdff}$ (||)

$$\frac{+fff - cff + dcf - dff}{f} = \frac{+fff - cff + cdf - dff}{b}$$

Ergo . . . . $f = b.$ Quod est contra Lemmatis hypothesin.

Non est igitur $f = a.$ vt erat positum. Quod de alia quacunque ex simili deductione demonstrandum est.

PROPOSITIO 23.

Æquationis

$$\begin{array}{l} aaaa - baaa + bcaa \\ \quad - caaa - bdaa \\ \quad + daaa - bfaa + bcda \\ \quad + faaa - cdaa + bcfa \\ \qquad\quad - cfaa - bdfa \\ \qquad\quad + dfaa - cdfa = - bcdf. \end{array}$$

est radix *b.* vel *c.* radici quæsititiæ *a.* æqualis.

Nam si propositæ æquationis radici *a.* ponatur *b.* æqualis, mutata *a.* in *b.* erit

$$\begin{array}{lrrr} \text{erit} & bbbb & -bbbb & +bcbb \\ & & -cbbb & -bdbb \\ & & +dbbb & -bfbb+bcdb \\ & & +fbbb & -cdbb+bcfb \\ & & & -cfbb-bdfb \\ & & & +dfbb-cdfb = -bcdf. \end{array}$$

Æqualitas autem ista reiectis contradictorijs manifesta est.

Ergo radici $a.$ posita $b.$ æqualis, æqualis est.

Item si propositæ æquationis radici $a.$ ponatur $c.$ æqualis, mutata $a.$ in $c.$

$$\begin{array}{lrrr} \text{erit} \ldots & +cccc & -bccc & +bccc \\ & & -cccc & -bdcc \\ & & +dccc & -bfcc+bcdc \\ & & +fccc & -cdcc+bcfc \\ & & & -cfcc-bdfc \\ & & & +dfcc-cdfc = -bcdf. \end{array}$$

Æqualitas autem ista reiectis contradictorijs manifesta est.

Ergo radici $a.$ posita $c.$ æqualis, æqualis quoque est.

Sunt igitur radices $b.$ & $c.$ radici quæsititiæ $a.$ æquales, vt est enunciatum.

Quod autem non detur radix alia præter $b.$ vel $c.$ æquationis radici $a.$ æqualis in sequenti Lemmate demonstratur.

Lemma.

Si dari possit radix aliqua æquationis radici $a.$ æqualis, quæ radicibus $b.$ vel $c.$ inæqualis sit, esto illa primum $d.$ vel $f.$

$$\begin{array}{lrrr} \text{Posito igitur } d = a. \text{ erit} \ldots & dddd & -bddd & +bcdd \\ & & -cddd & -bddd \\ & & +dddd & -bfdd+bcdd \\ & & +fddd & -cddd+bcfd \\ & & & -cfdd-bfdd \\ & & & +fddd-cfdd = -bcdf \end{array}$$

$$\text{Ergo} \ldots \quad \begin{array}{c} +2.dddd-2.cddd+2.fddd-2.cfdd. \\ \Vert \\ +2.bddd-2.bcdd+2.bfdd-2.bcdf \end{array}$$

$$\text{Hoc est} \ldots \quad \begin{array}{c} +bddd-bcdd+bfdd-bcdf \\ \Vert \\ +dddd-cddd+fddd-cfdd \end{array}$$

$$\text{Ergo} \ldots \quad \frac{ddd-cdd+fdd-cfd}{d} = \frac{+ddd-cdd+fdd-cfd}{b}$$

Ergo ..... $d = b.$ quod est contra Lemmatis hypothesim.

In similem inciditur contradictionem ex illata $d = c.$ si sedecim æquationis particularia pro $c.$ similiter ordinata fuerint.

Non est igitur $d = a.$ vt erat positum. Quod de $f.$ quoque vel alia quacunq; præter $b.$ $c.$ ex simili deductione pronuntiandum est.

PRO-

PROPOSITIO 24.

Æquationis

$$\begin{array}{llll} aaaa & -baaa & +bcaa & \\ & -caaa & +bdaa & \\ & -daaa & +bfaa & -bcda \\ & -faaa & +cdaa & -bcfa \\ & & +cfaa & -bdfa \\ & & +dfaa & -cdfa = -bcdf. \end{array}$$

sunt radices b. vel c. vel d. vel f. radici quæsititiæ a. æquales.

Nam si propositæ æquationis radici a. ponatur b æqualis, mutata a. in b. erit

$$\begin{array}{llll} bbbb & -bbbb & +bcbb & \\ & -cbbb & +bdbb & \\ & -dbbb & +bfbb & -bcdb \\ & -fbbb & +cdbb & -bcfb \\ & & +cfbb & -bdfb \\ & & +dfbb & -cdfb = -bcdf. \end{array}$$

Æqualitas autem ista reiectis contradictorijs manifesta est.

Ergo radici a. posita b. æqualis, æqualis est.

Item si propositæ æquationis radici a. ponatur c. æqualis, mutata a. in c. erit

$$\begin{array}{llll} cccc & -bccc & +bccc & \\ & -cccc & +bdcc & \\ & -dccc & +bfcc & -bcdc \\ & -fccc & +cdcc & -bcfc \\ & & +cfcc & -bdfc \\ & & +dfcc & -cdfc = -bcdf \end{array}$$

Æqualitas etiam ista reiectis contradictorijs manifesta est.

Ergo radici a. posita c. æqualis, æqualis quoque est.

Item positis d. vel f. radici a. æqualibus similes ex mutatione sequuntur æqualitates.

Vnde radices quoque illas radici a. æquales esse similiter est concludendum.

Sunt igitur radices b. c. d. f. radici quæsititiæ a. æquales, vt est enunciatum.

Radicem aliam præter b. c. d. f. æquationis radici quæsititiæ a. æqualem dari non posse, in sequenti Lemmate demonstratur.

Lemma.

Si dari possit radix aliqua æquationis radici a. æqualis quæ radicibus b. c. d. f. inæqualis sit, esto illa g. siue alia quæcunque.

Posito

Posito igitur $g = a$ erit
$$\begin{aligned} gggg &- bggg + bcgg \\ &- cggg + bdgg \\ &- dggg + cdgg - bcdg \\ &- fggg + bfgg - bcfg \\ &\qquad\;\; + cfgg - bdfg \\ &\qquad\;\; + dfgg - cdfg = -bcdf. \end{aligned}$$

Ergo ∴
$$gggg - cggg + cdgg - dggg + cfgg - fggg + dfgg - cdfg$$
$$=$$
$$bggg - bcgg + bcdg - bdgg + bcfg - bfgg + bdfg - bcdf$$

Ergo ∴
$$(ggg - cgg + cdg - dgg + cfg - fgg + dfg - cdf)\,g$$
$$=$$
$$(ggg - cgg + cdg - dgg + cfg - fgg + dfg - cdf)\,b$$

Ergo ∴ $g = b$. Quod est contra Lemmatis hypothesim.

In similem inciditur contradictionem ex illata $g = c$. vel $g = d$. vel $g = f$. Si 16. æquationis particularia simili deductioni pro c. d. f. apposite ordinata fuerint. Sed sufficiat pro exemplo quod iam de b. probatum est ad positionis falsitatem in reliquis redarguendam.

Non est igitur $g = a$. vt erat positum, quod de alia quacunq; ex deductionis paritate pronunciandum est.

Reductitiæ.

PROPOSITIO 25.

Æquationis
$$\begin{aligned} aaaa &- bbaa + bbca \\ &- ccaa + bbda \\ &- ddaa + bcca \\ &- bcaa + ccda \\ &- bdaa + bdda \\ &- cdaa + cdda = + bbcd \\ &\qquad\;\; + 2.bcda \quad + bccd \\ &\qquad\qquad\qquad\quad\;\; + bcdd. \end{aligned}$$
est radix b. vel c. vel d. radici quæsititiæ a. æqualis.

Nam si propositæ æquationis radici a. ponatur b. æqualis, mutata a. in b. erit

$$
\begin{array}{lllcl}
\text{erit} \; \ldots\ldots & bbbb - bbbb + bbcb & & & \\
 & \quad - ccbb + bbdb & & & \\
 & \quad - ddbb + bccb & & & \\
 & \quad - bcbb + ccdb & & & \\
 & \quad - bdbb + bddb & & & \\
 & \quad - cdbb + cddb & & & \\
 & \qquad + 2.bcdb & = & + bbcd & \\
 & & & + bccd & \\
 & & & + bcdd. &
\end{array}
$$

Æqualitas autem iſta reiectis contradictorijs manifeſta eſt.
Ergo radici $a.$ poſita $b.$ æqualis, æqualis eſt.
Item ſi propoſitæ æquationis radici $a.$ ponatur $c.$ æqualis, mutata $a.$ in $c.$

$$
\begin{array}{llcl}
\text{erit} \; \ldots\ldots & cccc - bbcc + bbcc & & \\
 & \quad - cccc + bbdc & & \\
 & \quad - ddcc + bccc & & \\
 & \quad - bccc + ccdc & & \\
 & \quad - bdcc + bddc & & \\
 & \quad - cdcc + cddc & & \\
 & \qquad + 2.bcdc & = & + bbcd \\
 & & & + bccd \\
 & & & + bcdd
\end{array}
$$

Æqualitas autem iſta ſeparatis contradictorijs manifeſta eſt.
Ergo radici $a.$ poſita $c.$ æqualis, æqualis eſt.
Item ſi propoſitæ æquationis radici $a.$ ponatur $d.$ æqualis, mutata $a.$ in $d.$ erit

$$
\begin{array}{llcl}
\ldots\ldots & dddd - bbdd + bbcd & & \\
 & \quad - ccdd + bbdd & & \\
 & \quad - dddd + bccd & & \\
 & \quad - bcdd + ccdd & & \\
 & \quad - bddd + bddd & & \\
 & \quad - cddd + cddd & & \\
 & \qquad + 2.bcdd & = & + bbcd \\
 & & & + bccd \\
 & & & + bcdd
\end{array}
$$

Æqualitas etiam iſta reiectis contradictorijs manifeſta eſt.
Ergo radici $a.$ poſita $d.$ æqualis, æqualis quoque eſt.
Sunt igitur $b.\, c.\, d.$ radices, radici quæſititiæ $a.$ æquales, vt eſt enunciatum.

PROPOSITIO 26.

$$
\begin{array}{lllcl}
\text{Æquationis} & aaaa - bbaaa & + bbcca & & \\
 & \quad - ccaaa & + bbdda & & \\
 & \quad - ddaaa & + ccdda & & \\
 & \quad - bcaaa & + bcdda & & \\
 & \quad - bdaaa & + bccda & & \\
 & \quad - cdaaa & + bbcda & = & + bbccd \\
 & \quad \overline{b+c+d} & \overline{b+c+d} & & + bbcdd \\
 & & & & + bccdd \\
 & & & & \overline{b+c+d.}
\end{array}
$$

eſt radix $b.$ vel $c.$ vel $d.$ radici quæſititiæ $a.$ æqualis.

 Nam

Nám si propositæ æquationis radici a. ponatur b. æqualis, mutata a. in b. & potestate ad communem diuisorem reducta erit . . . .

$$\frac{\begin{array}{l}+bbbbb\\+cbbbb\\+dbbbb\end{array}}{b+c+d}\;\frac{\begin{array}{l}-bbbbb\\-ccbbb\\-ddbbb\\-bcbbb\\-bdbbb\\-cdbbb\end{array}}{b+c+d}\;\frac{\begin{array}{l}+bbccb\\+bbcdb\\+ccddb\\+bcddb\\+bccdb\\+bbcdb\end{array}}{b+c+d} = \frac{\begin{array}{l}+bbccd\\+bbcdd\\+bccdd\end{array}}{b+c+d}$$

Æqualitas autem ista separatis contradictorijs manifesta est.

Ergo radici a. posita b. æqualis, æqualis est.

Item positis c. vel d. radici a. æqualibus similes ex mutatione sequuntur æqualitates.

Vnde radices quoque illas radici a. æquales esse, similiter est concludendum.

Sunt igitur radices b. c. d. radici quæsititiæ a. æquales, vt est enunciatum.

PROPOSITIO 27.

Æquationis

$$aaaa\;\frac{\begin{array}{l}-bbcaaa\\-bbdaaa\\-bccaaa\\-bddaaa\\-ccdaaa\\-cddaaa\\-2.bcdaaa\end{array}}{bc+bd+cd}\;\frac{\begin{array}{l}+bbccaa\\+bbddaa\\+ccddaa\\+bcddaa\\+bccdaa\\+bbcdaa\end{array}}{bc+bd+cd} = \frac{+bbccdd}{bc+bd+cd}$$

est radix b. vel c. vel d. radici quæsititiæ a. æqualis.

Nam si propositæ æquationis radici a. ponatur b. æqualis, mutata a. in b. & potestate ad communem diuisorem reducta erit . . .

$$\frac{\begin{array}{l}+bcbbbb\\+bdbbbb\\+cdbbbb\end{array}}{bc+bd+cd}\;\frac{\begin{array}{l}-bbcbbb\\-bbdbbb\\-bccbbb\\-bddbbb\\-ccdbbb\\-cddbbb\\-2.bcdbbb\end{array}}{bc+bd+cd}\;\frac{\begin{array}{l}+bbccbb\\+bbddbb\\+ccddbb\\+bcddbb\\+bccdbb\\+bbcdbb\end{array}}{bc+bd+cd} = \frac{+bbccdd}{bc+bd+cd}$$

Æqualitas

Æqualitas autem iſta ſeparatis contradictorijs manifeſta eſt.

Ergo radici a. poſita b. æqualis, æqualis eſt.

Item poſitis c. vel d. radici a. æqualibus ſimiles ex mutatione ſequuntur æqualitates.

Vnde radices quoque illas radici a. æquales eſſe ſimiliter eſt concludendum.

Sunt igitur radices b. c. d. radici quæſititiæ a. æquales, vt eſt enunciatum.

PROPOSITIO 28.

Si ſit

$$aaaa \begin{array}{l} -bbaa\\ -ccaa\\ -ddaa\\ -bcaa\\ -bdaa\\ -cdaa\\ \end{array} \begin{array}{l} -bbca\\ -bbda\\ -bcca\\ -ccda\\ -bdda\\ -cdda\\ -2.bcda \end{array} = \begin{array}{l} +bbcd\\ +bccd\\ +bcdd. \end{array}$$

eſt radix $b+c+d$. radici quæſititiæ a. æqualis.

Nam (per 12. Problem. Sect. 3.) æquatio trinomia hic propoſita à quadrinomia ſua deducitur poſito $b+c+d = f$.

Sed (per 21. Propoſ. huius) eſt quadrinomiæ illius radix $a = f$.

Eſt igitur trinomiæ huius radix $a = b+c+d$. vt eſt enunciatum.

PROPOSITIO 29.

Æquationis

$$aaaa \begin{array}{l} +bbaaa\\ +ccaaa\\ +ddaaa\\ +bcaaa\\ +bdaaa\\ +cdaaa\\ \hline b+c+d \end{array} \begin{array}{l} -bbcca\\ -bbdda\\ -ccdda\\ -bcdda\\ -bccda\\ -bbcda\\ \hline b+c+d \end{array} = \begin{array}{l} +bbccd\\ +bbcdd\\ +bccdd\\ \hline b+c+d \end{array}$$

eſt $\dfrac{bc+bd+cd}{b+c+d}$ radix radici quæſititiæ a. æqualis.

Nam (per 13. Probl. Sect. 3.) æquatio trinomia hic propoſita à quadrinomia ſua deducitur

citur posito $\dfrac{bc+bd+cd}{b+c+d} = f$.

Sed (per Prop. 21. huius) est quadrinomiæ illius radix $a = f$.

Est igitur trinomiæ huius radix $a = \dfrac{bc+bd+cd}{b+c+d}$. vt est enunciatum.

PROPOSITIO 30.

Æquationis
$$\begin{array}{lllll} aaaa+bbcaaa & & & & \\ \quad +bbdaaa & +bbccaa & & & \\ \quad +bccaaa & +bbddaa & & & \\ \quad +bddaaa & +ccddaa & & & \\ \quad +ccdaaa & +bcddaa & & & \\ \quad +cddaaa & +bccdaa & & & \\ \quad +2bcdaaa & +bbcdaa & = & +bbccdd & \\ \hline \quad bc+bd+cd & bc+bd+cd & & bc+bd+cd & \text{est} \end{array}$$

radix $\dfrac{bcd}{bc+bd+cd}$. radici quæsititiæ a. æqualis.

Nam (per 14. Probl. Sect. 3. æquatio trinomia hic proposita à quadrinomia sua deducitur, posito $\dfrac{bcd}{bc+bd+cd} = f$.

Sed (per 22. Prop. Sect. huius) est quadrinomiæ illius radix $a = f$.

Est igitur trinomiæ huius radix $a = \dfrac{bcd}{bc+bd+cd}$ vt est enunciatum.

PROPOSITIO 31.

Æquationis
$$\begin{array}{llll} aaaa+bdaa & +bbca & & \\ \quad +cdaa & +bcca & & \\ \quad -bbaa & +bdda & & \\ \quad -bcaa & +cdda & & \\ \quad -ccaa & -bbda & & \\ \quad -ddaa & -ccda & & \\ & -2bcda & = & -bbcd \\ & & & -bccd \\ & & & +bcdd, \end{array}$$
est radix b. vel c. radici quæsititiæ a. æqualis.

Nam

Nam si propositæ æquationis radici a. ponatur b. æqualis, mutata a. in b. erit . . . .

$$\begin{array}{llll}
bbbb + bdbb + bbcb & & & \\
+ cdbb + bccb & & & \\
- bbbb + bddb & & & \\
- bcbb + cddb & & & \\
- ccbb - bbdb & & & \\
- ddbb - ccdb & & & \\
\quad - 2bcdb & = & - bbcd & \\
 & & - bccd & \\
 & & + bcdd &
\end{array}$$

Æqualitas autem ista separatis contradictorijs manifesta est.

Ergo radici a. posita b. æqualis, æqualis est.

Item posita c. radici a. æquali, similis ex mutatione sequitur æqualitas.

Vnde radicem quoque illam radici a. æqualem esse, similiter est concludendum.

Sunt igitur radices b. c. radici quæsititiæ a. æquales, vt est enunciatum.

PROPOSITIO 32.

Æquationis

$$\begin{array}{llll}
aaaa - bbaaa + bbcca & & & \\
- bcaaa + bbdda & & & \\
- ccaaa + bcdda & & & \\
- ddaaa + ccdda & & & \\
+ bdaaa - bbcda & & & \\
+ cdaaa - bccda & = & - bbccd & \\
\overline{b + c - d} \quad \overline{b + c - d} & & + bbcdd & \\
 & & + bccdd & \\
 & & \overline{b + c - d} & \text{est radix}
\end{array}$$

b. vel c. radici quæsititiæ a. æqualis.

Nam si propositæ æquationis radici a. ponatur b. æqualis, mutata a. in b. & mutatæ potestatis ad communem diuisorem reductione factâ, erit . . .

$$\begin{array}{llll}
+ bbbbb - bbbbb + bbccb & & & \\
+ cbbbb - bcbbb + bbddb & & & \\
- dbbbb - ccbbb + bcddb & & & \\
\overline{b + c - d} - ddbbb + ccddb & & & \\
\quad + bdbbb - bbcdb & & & \\
\quad + cdbbb - bccdb & = & - bbccd & \\
\quad \overline{b + c - d} \quad \overline{b + c - d} & & + bbcdd & \\
 & & + bccdd & \\
 & & \overline{b + c - d} &
\end{array}$$

Æqualitas autem ista separatis contradictorijs manifesta est.

Ergo radici a. posita b. æqualis, æqualis est.

Item posita $c.$ radici $a.$ æquali, similis ex mutatione sequitur æqualitas.

Vnde radicem quoque illam radici $a.$ æqualem esse, similiter concludendum est.

Sunt igitur radices $b.$ $c.$ radici quæsititiæ $a.$ æquales, vt est enunciatum.

PROPOSITIO 33.

Æquationis
$$aaaa+\frac{\begin{array}{l} bbaaa\\ +bcaaa\\ +ccaaa\\ +ddaaa\\ -bdaaa\\ -cdaaa\end{array}}{d-b-c}\frac{\begin{array}{l} -bbcca\\ -bbdda\\ -bcdda\\ -ccdda\\ +bbcda\\ +bccda\end{array}}{d-b-c} = \frac{\begin{array}{l} -bbcdd\\ -bccdd\\ +bbccd\end{array}}{d-b-c}$$
est radix $b.$ vel $c.$ radici quæsititiæ $a.$ æqualis.

Nam si propositæ æquationis radici $a.$ ponatur $b.$ æqualis, mutata $a.$ in $b.$ & mutatæ potestatis ad communem diuisorem reductione factâ, erit ....
$$\frac{\begin{array}{l} +dbbbb\\ -bbbbb\\ -cbbbb\end{array}}{d-b-c}\frac{\begin{array}{l} +bbbbb\\ +bcbbb\\ +ccbbb\\ +ddbbb\\ -bdbbb\\ -cdbbb\end{array}}{d-b-c}\frac{\begin{array}{l} -bbccb\\ -bbddb\\ -bcddb\\ -ccddb\\ +bbcdb\\ +bccdb\end{array}}{d-b-c} = \frac{\begin{array}{l} -bbcdd\\ -bccdd\\ -bbccd\end{array}}{d-b-c}$$

Æqualitas autem ista separatis contradictorijs manifesta est.

Ergo radici $a.$ posita $b.$ æqualis, æqualis est.

Item posita $c.$ radici $a.$ æquali, similis ex mutatione sequitur æqualitas.

Vnde radicem quoque illam radici $a.$ æqualem esse, similiter est concludendum.

Sunt igitur radices $b.$ $c.$ radici quæsititiæ $a.$ æquales, vt est enuntiatum.

PROPOSITIO 34.

Æquationis
$$aaaa+\frac{\begin{array}{l} bbcaaa\\ +bccaaa\\ +bddaaa\\ +cddaaa\\ -bbdaaa\\ -ccdaaa\\ -2.bcdaaa\end{array}}{bd+cd-bc}\frac{\begin{array}{l} -bbccaa\\ -bbddaa\\ -bcddaa\\ -ccddaa\\ +bccdaa\\ +bccdaa\end{array}}{dc+cd+bc} = \frac{-bbccdd}{bd+cd-bc}$$
est radix $b.$ vel $c.$ radici quæsititiæ $a.$ æqualis.

Nam

Nam si propositæ æquationis radici a. ponatur b. æqualis, mutatâ a in b. & mutatæ potestatis reductione ad communem diuisorem factâ,

erit . . .

$$\begin{array}{lll}
+bdbbbb & +bbcbbb & \\
+cdbbbb & +bccbbb & -bbccbb \\
-bcdbbb & +bddbbb & -bbddbb \\
\overline{bd+cd-bc} & -cddbbb & -bcddbb \\
 & -bbdbbb & -ccddbb \\
 & -ccdbbb & +bbcdbb \\
 & -2.bcdbbb & +bccdbb = -bbccdd \\
 & \overline{bd+cd-bc} & \overline{bd+cd-bc} \qquad \overline{bd+cd-bc}
\end{array}$$

Æqualitas autem ista separatis contradictorijs manifesta est.

Ergo radici a. posita b. æqualis, æqualis est.

Item posita c. radici a. æquali similis ex mutatione sequitur æqualitas.

Vnde radicem quoq; illam radici a. æqualem esse, similiter concludendum est.

Sunt igitur radices b. c. radici quæsititiæ a. æquales, vt est enunciatum.

PROPOSITIO 35.

Æquationis

$$\begin{array}{l}
aaaa-bbba \\
\quad -bbca \\
\quad -bcca \\
\quad -ccca = -bbbc \\
\qquad\qquad -bbcc \\
\qquad\qquad -bccc.
\end{array}$$

est radix b. vel c. radici quæsititiæ a. æqualis.

Nam posito $b = a$. erit . . .

$$\begin{array}{l}
bbbb-bbbb \\
\quad -bbbc \\
\quad -bbcc \\
\quad -bccc = -bbbc \\
\qquad\qquad -bbcc \\
\qquad\qquad -bccc
\end{array}$$

Vel posito $c = a$. erit . . .

$$\begin{array}{l}
cccc-bbbc \\
\quad -bbcc \\
\quad -bccc \\
\quad -cccc = -bbbc \\
\qquad\qquad -bbcc \\
\qquad\qquad -bccc
\end{array}$$

Æqualitates manifestæ sunt.

Est igitur æquationis propositæ radix $a = b$. vel c. vt est enunciatum.

PRO-

PROPOSITIO 36.

Æquationis $$aaaa\,\frac{-bbbaaa-bbcaaa-bccaaa-cccaaa}{bb+bc+cc} = \frac{-bbbccc}{bb+bc+cc}$$ est radix b. vel c. radici quæsititiæ a. æqualis.

Nam posito $b = a$. erit . . . $$\frac{+bbbbbb+bbbbbc+bbbbcc}{bb+bc+cc}\,\frac{-bbbbbb-bbbbbc-bbbbcc-bbbccc}{bb+bc+cc} = \frac{-bbbccc}{bb+bc+cc}$$

Vel posito $c = a$. erit $$\frac{+bbcccc+bccccc+cccccc}{bb+bc+cc}\,\frac{-bbbccc-bbcccc-bccccc-cccccc}{bb+bc+cc} = \frac{-bbbccc}{bb+bc+cc}$$

Æqualitates manifestæ sunt.

Est igitur æquationis propositæ radix $a = b$. vel c. vt est enunciatum.

PROPOSITIO 37.

Æquationis $$aaaa - bbaa - ccaa = -bbcc,$$ est radix b. vel c. radici quæsititiæ a. æqualis.

Nam posito $b = a$. erit . . . . $bbbb - bbbb - bbcc = -bbcc$

Vel posito $c = a$. erit $cccc - bbcc - cccc = -bbcc$

Æqualitates manifestæ sunt.

Est igitur æquationis propositæ radix $a = b$. vel c. vt est enunciatum.

Reciprocæ.

PROPOSITIO 38.

Æquationis $aaaa - baaa + cdfa = +bcdf$. est b. radix radici quæsititiæ a. æqualis.

Nam

Nam si æquationis $aaaa - baaa + cdfa = + bcdf$. radici a. ponatur b. æqualis, mutata a. in b. erit $bbbb - bbbb + cdfb = + cdfb$.

Est autem æqualitas ista per se manifesta.

Ergo radici a. posita b. æqualis, æqualis est, vt est enunciatum.

PROPOSITIO 39.

Æquationis $aaaa + baaa - ccca = + bccc$. est c. radix radici quæsititiæ a. æqualis.

Nam si æquationis $aaaa + baaa - ccca = + bccc$. radici a. ponatur c. æqualis, mutata a. in c. erit $cccc + bccc - cccc = + bccc$.

Est autem æqualitatis huius veritas per se manifesta.

Ergo radici a. posita c. æqualis, æqualis est, vt est enunciatum.

PROPOSITIO 40.

Æquationis $aaaa - baaa - ccca = - bccc$. sunt b. vel c. radices radici quæsititiæ a. æquales.

Nam si æquationis $aaaa - baaa - ccca = - bccc$. radici a. ponatur b. æqualis, mutata a. in b. erit $bbbb - bbbb - cccb = - bccc$.

Est autem æqualitatis huius veritas per se manifesta.

Ergo radici a. posita b. æqualis, æqualis est.

Item si radici a. ponatur c. æqualis, mutata a. in erit $cccc - bccc - cccc = - bccc$.

Est autem æqualitatis huius veritas manifesta.

Ergo radici a posita c. æqualis, æqualis est.

Sunt igitur b. vel c. radices radici quæsititiæ a. æquales, vt ennnciatum.

Sectio quinta in qua æquationum communium per canonicarum æquipollentiam, radicum numerus determinatur.

DEFINITIO.

DVæ æquationes similiter graduatæ & similiter affectæ, quarum coefficiens vel coefficientia (si plura sint) & homogeneum datū vnius coefficienti vel coefficientibus & homogeneo dato alterius in simplici inæqualitatis, maioritatis scilicet & minoritatis habitudine conformia sunt, æquipollentes in sequentibus appellandæ sunt. Quod sic rursus interpretandum est, quasi æquali radicum numero pollentes. Hinc est quod æquationibus è radicibus binomijs generatis & earum reductitijs, de quibus in superioribus tribus Sectionibus tractatum est, Canonicarum nomen impositum est: quia factâ earum ad æquationes communes comparatione, si supradictis æquipollentiæ conditionibus inter se conueniant, ad radicum numerum in æquationibus communibus dignoscendum & determinandum canones siue exemplaria certa & solennia sint. In conformitate igitur inter æquationum communium & canonicarum coefficientia & homogenea data instituendâ, æquationum communium coefficientia & homogenea formali canonicarum partitioni similiter partienda sunt, & similes vtrinque partes sumendæ, seruatâ in partium habitudine æstimandâ homogeniæ lege, per reductionem scilicet procuratâ homogeniâ; cum coefficientia & homogenea data necessariò heterogenea sint, & de heterogeneorum inter se habitudine nulla fieri possit æstimatio.

Lemma 1.

Si quantitas secetur in duas partes inæquales quadratum è dimidia totius maior est facto è duabus partibus inæqualibus.

Si sint p. & q. duæ magnitudinis partes inæquales,
est . . . . $\frac{p+q}{2} \cdot \frac{p+q}{2} > pq$.

Nam è tribus continuè proportionalibus pp. pq. qq. quarum pq. maxima est, cc. verò minima, est . . . $pp - pq > pq - qq$.

Ergo . . . . $pp + qq > 2.pq$.

Et addito vtrinq; $2.pq$. est . . $pp + 2.pq + qq > 4.pq$

Sed . . . . $pp + 2.pq + qq = (p+q)(p+q)$

Ergo . . . . $(p+q)(p+q) > 4.pq$.

Ergo . . . . $\frac{(p+q)(p+q)}{4} > pq$.

Ergo . . . . $\left.\begin{array}{c}\frac{p+q}{2}\\ \frac{p+q}{2}\end{array}\right\} > pq.$

Quod erat probandum.

Lemma 2.

Si fuerint tres continuè proportionales summa extremarum maior est bis media.

Si sint $b.\ c.\ d.$ continuè proportionales, est $b+d > 2.\ c.$

Nam si sit $b.$ maxima, erit . . . . : $b-c > c-d.$

Ergo . . . . $b+d > 2.\ c.$

Vel si $d.$ maxima, erit . . . . $d-c > c-b.$

Ergo . . . . $d+b > 2.\ c.$

Est igitur summa extremarum maior bis media, vt est enunciatum.

Lemma 3.

Si fuerint quatuor continuè proportionales summa extremarum maior est summa mediarum.

Si sint $b.\ c.\ d.\ f.$ continuè proportionales, est $b+f > c+d.$

Nam si $b.$ maxima sit, erit . . . . . . $b-c > d-f.$

Ergo . . . . $b+f > c+d.$

Vel si $f.$ maxima, erit . . . . $f-d > c-b.$

Ergo . . . . $f+b > c+d.$

Est igitur summa extremarum maior quam summa mediarum, vt est enunciatum.

PROPOSITIO I.

Æquatio communis $aaa-3.bba = +2.ccc.$ in qua $c > b.$ de simplici radice explicabilis est.

Nam æquatio communis proposita æquationi Canonicæ. $aaa-3.rqa = \begin{array}{l}+rrr\\+qqq\end{array}$ similiter graduata & similiter affecta est.

Et

Et (per Lemma 4. sequens) in æquatione canonica est. . . $\left.\begin{array}{l} rq \\ rq \\ rq \end{array}\right| < \dfrac{\left.\begin{array}{l} rrr+qqq \\ rrr+qqq \end{array}\right|}{4}$

Atque in æquatione proposita in qua supponitur $b < c$.
est $bbbbbb < cccccc$.

Ergo coefficiens & homogeneum datum propositæ, coefficienti & homogeneo dato Canonicæ, in excessus & defectus habitudine conformia sunt.

Sunt igitur (per definitionem) æquatio proposita & Canonica æquipollentes, æquali scilicet radicum numero præditæ.

Sed (per Prop. 14. Sect 4) æquatio Canonica de simplici radici $q+r$. explicabilis est.

Explicabilis est igitur æquatio communis proposita de radici simplici, vt est enunciatum.

Lemma 4.

Est . . . . $\dfrac{\left.\begin{array}{l} rrr+qqq \\ rrr+qqq \end{array}\right|}{4} > \left.\begin{array}{l} rq \\ rq \\ rq \end{array}\right|$

Sunt enim . . . . $rrrrrr.\ rrrqqq.\ qqqqqq.$ continuè proportionales.

Ergo (per Lem. 2.) . . . . $rrrrrr+qqqqqq. > 2.rrrqqq.$

Et addito vtrinque $2.rrrqqq$.

Est . . . . . . $rrrrrr+2.rrrqqq+qqqqqq > 4.rrrqqq.$

Est autem . . . $rrrrrr+2.rrrqqq+qqqqqq = \left.\begin{array}{l} rrr+qqq \\ rrr+qqq \end{array}\right|$

Et . . . . . $4.rrrqqq = \left.\begin{array}{l} 4.rq \\ rq \\ rq \end{array}\right|$

Ergo . . . . $\left.\begin{array}{l} rrr+qqq \\ rrr+qqq \end{array}\right| > \left.\begin{array}{l} 4.rq \\ rq \\ rq \end{array}\right|$

Ergo . . . . . $\dfrac{\left.\begin{array}{l} rrr+qqq \\ rrr+qqq \end{array}\right|}{4} > \left.\begin{array}{l} rq \\ rq \\ rq \end{array}\right|$ Quod erat probandum.

PROPOSITIO 2.

Æquatio communis $aaa - 3.bba = +2.ccc$ in qua $c < b$ de radice simplici explicabilis est.

Nam

Nam æquatio communis proposita æquationi Canonicæ similiter graduata & similiter affecta est.

$$aaa \begin{array}{l} -qqa\\ -qra\\ -rra \end{array} = \begin{array}{l} +qqr\\ +qrr \end{array}$$

Et (per Lem.5. sequens) in æquatione Canonica est.

$$\frac{\begin{array}{l|} +qqr\\ +qrr \end{array}}{4} < \frac{\begin{array}{l|} qq+qr+rr\\ qq+qr+rr\\ qq+qr+rr \end{array}}{27}$$

Atque in æquatione proposita in qua supponitur $c < b$.
est .... $cccccc < bbbbbb$.

Ergo coefficiens & homogeneum datum propositæ, coefficienti & homogeneo dato canonicæ in inæqualitatis habitudine conformia sunt.

Sunt igitur (per def.) æquatio proposita & canonica æquipollentes, æquali scilicet radicum numero præditæ.

Sed (per Prop.7. Sect.4.) æquatio canonica de simplici radice $q+r$. explicabilis est.

Explicabilis est igitur æquatio communis proposita de radice simplici, vt est enunciatum.

Lemma 5.

Est ....

$$\frac{\begin{array}{l|} qqr+qrr\\ qqr+qrr \end{array}}{4} < \frac{\begin{array}{l|} qq+qr+rr\\ qq+qr+rr\\ qq+qr+rr\\ qq+qr+rr \end{array}}{27}$$

Nam per (Lem 3.) est .. $3.qqqqrr + 3.qqrrrr < 3.qqqqqq + 3.rrrrrr$.

Et (per Lem.2.) .. $12.qqqrrr + 12.qqqrrr < 12.qqqqqr + 12\,qrrrrr$.

Et (per Lem.2.) ... $qqqrrr + qqqrrr < qqqqqq + rrrrrr$.

Ergo .....

$$\begin{array}{l} +3.qqqqrr\\ +26.qqqrrr\\ +3.qqrrrr \end{array} < \begin{array}{l} +4.qqqqqq\\ +12.qqqqqr\\ +12.qrrrrr\\ +4.rrrrrr \end{array}$$

Ergo addito vtrinque ... $24.qqqqrr + 28.qqqrrr + 24.qqrrrr$.

Est ....

$$\begin{array}{l} +27.qqqqrr\\ +54.qqqrrr\\ +27.qqrrrr \end{array} < \begin{array}{l} +4.qqqqqq\\ +12\,qqqqqr\\ +12.qrrrrr\\ +4.rrrrrr\\ +24.qqqqrr\\ +28.qqqrrr\\ +24.qqrrrr \end{array}$$

Diuisis igitur particularibus vtriusque partis communiter per 4. & iteratò communiter per 27.

Est . . . . . $\dfrac{\begin{array}{l} +1.qqqqrr \\ +2.qqqrrr \\ +1.qqrrrr \end{array}}{4} < \dfrac{\begin{array}{l} +1.qqqqqq \\ +3.qqqqqr \\ +3.qrrrrr \\ +1.rrrrrr \\ +6.qqqqrr \\ +7.qqqrrr \\ +6.qqrrrr \end{array}}{27}$,

Sed . . . . . . $\dfrac{\begin{array}{l} +1.qqqqrr \\ +2.qqqrrr \\ +1.qqrrrr \end{array}}{4} = \dfrac{\begin{array}{l} +qqr+qrr| \\ +qqr+qrr| \end{array}}{4}$.

Et . . . . $\dfrac{\begin{array}{l} +1.qqqqqq \\ +3.qqqqqr \\ +3.qrrrrr \\ +1.rrrrrr \\ +6.qqqqrr \\ +7.qqqrrr \\ +6.qqrrrr \end{array}}{27} = \dfrac{\begin{array}{l} +qq+qr+rr| \\ +qq+qr+rr| \\ +qq+qr+rr| \end{array}}{27}$.

Ergo . . . . . . $\dfrac{\begin{array}{l} +qqr+qrr| \\ +qqr+qrr| \end{array}}{4} < \dfrac{\begin{array}{l} +qq+qr+rr| \\ +qq+qr+rr| \\ +qq+qr+rr| \end{array}}{27}$.

Quod erat demonstrandum.

PROPOSITIO 3.

Æquatio communis $aaa - 3.bba = +2.ccc$, in qua $c = b$, de simplici radice explicabilis est.

Nam Æquatio communis proposita æquationi canonicæ . . . . . $aaa - 3.qqa = + 2.qqq$. similiter graduata & similiter affecta est.

Et in æquatione canonica est $q = q$.

Atque in æquatione proposita supponitur $b = c$.

Ergo coefficiens & homogeneum datum propositæ coefficienti & homogeneo dato canonicæ in æqualitatis habitudine conformia sunt.

Sunt enim vtrinque æqualia, factâ ex homogeniæ lege vt oportet comparatione,

Sunt igitur æquatio proposita & canonica æquipollentes, æquali scilicet radicum numero præditæ.

Sed (per Prop. 17. Sect 4.) æquatio canonica de simplici radice $2.q$. explicabilis est.

Explicanda est igitur æquatio communis proposita de radice simplici, vt est enunciatum.

PRO-

PROPOSITIO 4.

Æquatio communis $aaa - 3.bba = -2.ccc.$ in qua $b > c.$ de duplici radice explicabilis est.

Nam æquatio communis proposita æquationi canonicæ . . . . . .

$$\begin{array}{l} aaa - qqa \\ \quad - qra \\ \quad - rra = - qqr \\ \qquad\qquad - qrr. \end{array}$$

similiter graduata & similiter affecta est.

Et (per Lem. 5. ad Prop. 2.) in æquatione canonica, est . . . . . .

$$\frac{\left.\begin{array}{l} qq + qr + rr \\ qq + qr + rr \\ qq + qr + rr \end{array}\right|}{27.} > \frac{\left.\begin{array}{l} + qqr \\ + qrr \end{array}\right|}{4.}$$

Atque in æquatione proposita in qua suponitur $b > c.$ est . . . . $bbbbbb > cccccc.$

Ergo coefficiens & homogeneum datum propositæ, coefficienti & homogeneo dato canonicæ in excessus & defectus habitudine conformia sunt.

Sunt igitur (per Def.) æquatio canonica & proposita æquipollentes, æquali scilicet radicum numero pollentes.

Sed (per Prop. 6. Sect. 4) æquatio canonica de duplici radice $q.$ & $r.$ explicabilis est.

Explicabilis est igitur æquatio proposita de radice duplici, vt est enunciatum.

PROPOSITIO 5.

Æquatio communis $aaa - 3baa + 3.cca = + ddd$ in qua $b > c.$ & $b > d.$ de radice triplici explicabilis est.

Nam æquatio communis proposita æquationi canonicæ

$$\begin{array}{l} aaa - paa + pqa \\ \quad - qaa + pra \\ \quad - raa + qra = + pqr \end{array}$$

similiter graduata, & similiter affecta est.

Et in æquatione canonica (per Lem. 6. sequens) est

$$\frac{\left.\begin{array}{l} \frac{p+q+r}{3.} \\ \frac{p+q+r}{3.} \end{array}\right|}{} > \frac{pq + pr + qr.}{3.}$$

Et

Et (per Lem. 7. sequens) $\frac{p+q+r}{3} \cdot \frac{p+q+r}{3} \cdot \frac{p+q+r}{3} > pqr$

Atque in æquatione proposita in qua supponitur $b > c.$ & $b > d.$ est $bb > cc.$ & $bbb > ddd.$

Ergo coefficentia & homogeneum datum propositæ coefficientibus & homogeneo dato canonicæ in excessus & defectus habitudine conformia sunt.

Sunt igitur (per Def.) æquatio proposita & canonica æquipollentes æquali scilicet radicum numero pollentes.

Sed (per Prop. 5. Sect. 4.) æquatio canonica de triplici radice $p.$ $q.$ $r.$ explicabilis est.

Explicabilis est igitur æquatio communis proposita de radice triplici, vt est enunciatum.

Lemma 6.

Si quantitas secetur in tres partes inæquales quadratum è tertia parte totius maius est tertia parte factorum è singulis binis inæqualibus.

Si sint quantitatis tres partes inequales $p.$ $q.$ $r.$ est

$$\frac{p+q+r}{3} \cdot \frac{p+q+r}{3} > \frac{pq+pr+qr.}{3}$$

Nam (per 2. Lem.) . . . . $pp+qq > 2.pq.$

Et . . . . $qq+rr > 2.qr.$

Et . . . . $pp+rr > 2.pr.$

Ergo . . . . $2.pp+2.qq+2.rr > 2.pq+2.qr+2.pr.$

Ergo . . . . $pp+qq+rr > pq+qr+pr.$

Addito igitur vtrinque . . . . $2.pq+2.qr+2.pr.$

Erit . . . . $\begin{matrix} pp+qq+rr \\ +2.pq+2.qr \\ +2.pr. \end{matrix} > 3.pq+3.qr+3.pr.$

Sed . . . . $\begin{matrix} pp+qq+rr \\ +2.pq+2.qr \\ +2.pr \end{matrix} = \overline{p+q+r} \cdot \overline{p+q+r}$

Ergo . . . . $\overline{p+q+r} \cdot \overline{p+q+r} > 3.pq+3.qr+3.pr.$

Hoc est . . . . $\frac{\overline{p+q+r} \cdot \overline{p+q+r}}{3} > pq+qr+pr.$

Ergo

Ergo . . . . $\left.\begin{array}{l}\frac{p+q+r}{3}\\ \frac{p+q+r}{3}\end{array}\right| > \frac{pq+qr+pr}{3}.$

Quod erat probandum.

Lemma 7.

Si quantitas secetur in tres partes inæquales Cubus è tertia parte totius maior est solido è tribus inæqualibus.

Si sint quantitatis tres partes inæquales $p.$ $q.$ $r.$ est

$$\left.\begin{array}{l}\frac{p+q+r}{3}\\ \frac{p+q+r}{3}\\ \frac{p+q+r}{3}\end{array}\right| > pqr.$$

Nam (per 2. Lem.) . . . . $pp+qq > 2.pq.$

Et . . . . $qq+rr > 2\,qr.$

Et . . . . $pp+rr > 2.pr.$

Ergo . . . . $ppr+qqr > 2.pqr.$

Et . . . . $pqq+prr > 2.pqr.$

Et . . . . $ppq+qrr > 2.pqr.$

Ergo . . . . $\left.\begin{array}{l}+pqq+prr\\ +ppq+qrr\\ +ppr+qqr\end{array}\right\} > 6.pqr$

Sed (per Lem. 3) . . . . $ppp+qqq > ppq+pqq$

Et . . . . $qqq+rrr > qqr+qrr$

Et . . . . $ppp+rrr > ppr+prr$

Ergo . . . . $ppp+qqq+rrr > 3.pqr$

Et . . . . $\left.\begin{array}{l}+3.pqq+3.prr\\ +3.ppq+3.qrr\\ +3.ppr+3.qqr\end{array}\right\} > 18.pqr.$

Ergo . . . . $\left.\begin{array}{l}ppp+qqq+rrr\\ +3.pqq+3.prr\\ +3.ppq+3.qrr\\ +3.ppr+3.qqr\end{array}\right\} > 21.pqr$

Et addito vtrinque $6.pqr.$ est . . . $\left.\begin{array}{l}ppp+qqq+rrr\\ +3.pqq+3.prr\\ +3.ppq+3.qrr\\ +3.ppr+3.qqr\\ \qquad +6.pqr\end{array}\right\} > 27.pqr.$

 Sed

$$\text{Sed} \;\ldots\ldots\; \begin{array}{r} +ppp+qqq+rrr \\ +3.pqq+3.prr+3.ppq \\ +3.qrr+3ppr+3.qqr \\ +6.pqr \end{array} = \left.\begin{array}{l} p+q+r \\ p+q+r \\ p+q+r \end{array}\right|$$

$$\text{Ergo} \;\ldots\ldots\; \left.\begin{array}{l} p+q+r \\ p+q+r \\ p+q+r \end{array}\right| > 27.pqr.$$

$$\text{Hoc est} \;\ldots\ldots\; \left.\begin{array}{l} p+q+r \\ p+q+r \\ p+q+r \end{array}\right| \Big/ 27. > pqr$$

$$\text{Ergo} \;\ldots\ldots\; \left.\begin{array}{l} \frac{p+q+r}{3.} \\ \frac{p+q+r}{3.} \\ \frac{p+q+r}{3.} \end{array}\right| > pqr$$ Quod erat demonſtrandum.

PROPOSITIO 6.

Æquatio communis $aaaa - 4.bbba = -3.cccc.$ in qua $b > c.$ de duplici radice explicabilis eſt.

Nam æquatio communis propoſita æquationi canonicæ,

$$\begin{array}{ll} aaaa & bbba \\ & -bbca \\ & -bcca \\ & -ccca \end{array} = \begin{array}{l} -bbbc \\ -bbcc \\ -bcc. \end{array}$$ ſimiliter graduata & ſimiliter affecta eſt.

Et in æquatione canonica biquadraticè factum de $\frac{bbb+bbc+bcc+ccc.}{4.}$ maius eſt quàm cubicè factum de $\frac{bbbc+bbcc+bccc.}{3.}$

Atque in æquatione propoſita in qua ſuponitur $b > c.$ biquadraticè factum de $\frac{bbb.}{4.}$ maius quam cubicè factum de $\frac{cccc.}{3.}$

Ergo coefficiens & homogeneum datum propoſitæ coefficienti & homogeneo dato canonicæ in exceſſus & defectus habitudine conformia ſunt.

Sunt igtur (per Def.) æquatio propoſita & canonica æquipollentes, æquali ſcilicet radicum numero pollentes.

Sed (per Prop. 35 Sect. 4.) æquatio canonica de duplici radice $b.$ vel $c.$ explicabilis eſt. Explicabilis eſt igitur æquatio propoſita de radice duplici, vt eſt enunciatum.

Æqua-

Æquationum communium reductio per gradus alicuius parodici exclusionem & radicis suppositititiæ mutationem.

Problema de æquationum radicibus multiplicandis, æquationibus, quarum reductiones in præsenti Sectione traduntur, reductioni præparandis accommodum.

Æquationis propositæ, radicem seruatâ comparationis æqualitate, per quemcunque numerum datum multiplicare.

Sit æquatio quadratica . . . . . . $aa + ba = cc$.

Est igitur . . . $\left.\begin{array}{c}1\\ aa\end{array}\right| + \left.\begin{array}{c}b\\ a\end{array}\right| = \left.\begin{array}{c}cc\\ 1\end{array}\right|$

Quia vero æquationis radix duplicanda est, multiplicentur primo tria illius homogenea per tres numeros proportionales in ratione dupla 1. 2. 4. ordinatim applicatos.

Vnde eueniet . . . . $\left.\begin{array}{c}1\\ 1\\ aa\end{array}\right| + \left.\begin{array}{c}2\\ b\\ a\end{array}\right| < \left.\begin{array}{c}4\\ cc\\ 1\end{array}\right|$ destructa scil. æqualitate.

Pro æqualitate igitur restituenda multiplicentur denuo æquationis homogenea ijsdem numeris proportionalibus reciprocè applicatis scilicet 4. 2. 1.

Hinc fiet . . . . . . $\left.\begin{array}{c}1\\ 1\\ aa\\ 4\end{array}\right| + \left.\begin{array}{c}2\\ b\\ a\\ 2\end{array}\right| = \left.\begin{array}{c}4\\ cc\\ 1\\ 1\end{array}\right|$

Sunt enim. . . . . . $\left.\begin{array}{c}1\\ 4\end{array}\right| = \left.\begin{array}{c}2\\ 2\end{array}\right| = \left.\begin{array}{c}4\\ 1\end{array}\right|$ æqualia.

Sit . . . . $2.\ a = c$.

Ergo . . . . $\left|\begin{array}{c}aa.\\ 4.\end{array}\right.$ & $\left|\begin{array}{c}a.\\ 2.\end{array}\right.$ & $\left|\begin{array}{c}1.\\ 1.\end{array}\right. = ee.$ & $e.$ & $1.$

Ergo his in illorum loca substitutis,

fit . . . $\left.\begin{array}{c}1\\ 1\\ ee\end{array}\right| + \left.\begin{array}{c}2\\ b\\ e\end{array}\right| = \left.\begin{array}{c}4\\ cc\\ 1\end{array}\right|$

Ergo . . . . $ee + 2.\ be = 4.cc$.

Sic igitur æquationis propositæ radix a. mutatâ $2.\ a.$ in $e.$ seruatâ interim æqualitate, duplicata est; vt erat imperatum,

Pro

Pro æquationis cubicæ radice multiplicanda.

Sit æquatio cubica . . . . $aaa + baa + cca = ddd.$ cuius radix $a.$ triplicanda sit.

Quoniam est . . . . $aaa + baa + cca = ddd.$

est . . . $\begin{array}{c|}1\\ aaa\end{array} + \begin{array}{c|}b\\ aaa\end{array} + \begin{array}{c|}cc\\ a\end{array} = \begin{array}{c|}ddd\\ 1\end{array}$

Quia verò æquationis radix triplicanda est, multiplicentur primo quatuor illius homogenea per 4. numeros proportionales in ratione tripla 1. 3. 9. 27. ordinatim applicatos.

Vnde eueniet . . . $\begin{array}{c|}1\\ 1\\ aaa\end{array} + \begin{array}{c|}3\\ b\\ aa\end{array} + \begin{array}{c|}9\\ cc\\ aa\end{array} < \begin{array}{c|}27\\ ddd\\ 1\end{array}$ destructa scil. æqualitate.

Pro æqualitate igitur restituenda multiplicentur denuo æquationis homogenea ijsdem numeris proportionalibus reciproce applicatis scilicet 27. 9. 3. 1.

Hinc fiet . . . . $\begin{array}{c|}1\\ 1\\ aaa\\ 27\end{array} + \begin{array}{c|}3\\ b\\ aa\\ 9\end{array} + \begin{array}{c|}9\\ cc\\ a\\ 3\end{array} = \begin{array}{|c}27.\\ ddd\\ 1\\ 1\end{array}$

Sunt enim . . . . $\begin{array}{c|}1\\ 27\end{array} = \begin{array}{c|}3\\ 9\end{array} = \begin{array}{c|}9\\ 3\end{array} = \begin{array}{c|}27\\ 1\end{array}$ æqualia.

Sit . . . . $3.a = e.$

Ergo . . . . $\begin{array}{c|}aaa\\ 27\end{array}$ & $\begin{array}{c|}aa\\ 9\end{array}$ & $\begin{array}{c|}a\\ 3\end{array}$ & $\begin{array}{c|}1\\ 1\end{array} = eee.$ & $ee.$ & $e.$ & $1.$

Ergo his in illorum loca substitutis, fit . . . . $\begin{array}{c|}1\\ 1\\ eee\end{array} + \begin{array}{c|}3.\\ b\\ ee\end{array} + \begin{array}{c|}9\\ cc\\ e\end{array} = \begin{array}{|c}27\\ ddd\\ 1\end{array}$

Ergo . . . . $eee + 3.bee + 9.cce = 27.ddd.$

Sic igitur æquationis propositæ radix $a.$ mutata $3.a.$ in $e.$ triplicata est; vt erat imparatum.

Consectarium.

Ex multiplicatione radicis sequitur muliplicatio coefficientis simplicis secundùm eandẽ multiplicitatis rationem: vt in superioribus exemplis, in æquatione quadratica, radice $a.$ duplicata, coefficiens $b.$ per numerum item binarium, & in æquatione cubica radice $a.$ triplicata, coefficiens $b.$ per numerum itidem ternarium multiplicatur. Adeo vt Problema si de coefficiente multiplicando conceptum & enunciatum esset conuerso tantum sensu huic æquipollens foret. Nam coefficientis multiplicatio radicis multiplicationem præsupponit. Est autem coefficientis multiplicatio ista consequentialis in æquationibus

quationibus resoluendis ad fractiones vbi opus fuerit tollendas, vel in æquationum reductionibus tractandis, vbi coefficientis numeri vel speciei indiuisibilitas obstat, ad fractionum implicationes præcaueadas, tantæ commoditatis, vt hoc solum nomine præcipuus huius artificij vsus est vid eatur.

PROBLEMA 1.

Æquationem $aaa - 3.baa = +ccc$. posito $a = c + b$. ad æquationem $eee - 3.bbe = \begin{array}{l} +ccc \\ +2.bbb. \end{array}$ vel posito $a = -e + b$. ad $eee - 3.bbe = \begin{array}{l} -ccc \\ -2.bbb. \end{array}$ æquationem impossibilem reducere.

Ponatur primo . . . . $e + b = a$.

Et fiat . . . . $ee + 2.be + bb = aa$.

$$\begin{array}{ll} \text{Et} \;.\;.\;.\;. & eee + 3.bee + 3.bbe + bbb = +aaa \\ \text{Fiat quoque} \;.\;. & -3.bee - 6.bbe - 3.bbb = -3baa \end{array} \Big\} = +ccc.$$

Hinc reiectis contradictorijs & ordinatis reliquis fit . . . . $eee - 3.bbe = \begin{array}{l} +ccc \\ +2.bbb. \end{array}$ æquatio requisita prima cuius radix. $e = a - b$.

Ponatur secundo . . . . $-e + b = a$.

Et fiat . . . . $ee - 2.be + bb = aa$.

$$\begin{array}{ll} \text{Et} \;.\;.\;.\;. & -eee + 3.bee - 3.bbe + .bbb = +aaa \\ \text{Fiat quoque} \;.\;.\;.\;. & -3.bee + 6.bbe - 3.bbb = -3.baa \end{array} \Big\} + ccc.$$

Hinc reiectis contradictorijs & ordinatis reliquis.

fit . . . . . . $eee - 3.bbe = \begin{array}{l} -ccc \\ -2.bbb. \end{array}$ æquatio requisita secunda quam impossibilem esse in seqnenti Lemmate demonstratur.

Atque sic facta est propositæ æquationis ad requisitas imperata reductio.

Lemma.

Æquatio reductitia $eee - 3.bbe = \begin{array}{l} -ccc \\ -2.bbb. \end{array}$ impossibilis est.

Nam posito $e = b$. erit $bbb - 3.bbb = \begin{array}{l} -ccc \\ -2.bbb. \end{array}$

Ergo . . $ccc = 0$. quod est impossibile.

Vel posito $e > b$. hoc est $e = b + d$

erit . . .
$$\begin{aligned}&+bbb\\&+3.bbd\\&+3.bdd-3.bbb\\&+ddd-3.bbd = -ccc\\&\qquad\qquad\qquad -2.bbb\end{aligned}$$

Ergo separatis contradictorijs; est $\begin{aligned}&+3.bdd\\&+.ddd = -ccc.\end{aligned}$

Ergo . . . $\begin{aligned}&+3.bdd\\&+ddd\\&+ccc = 0.\end{aligned}$ quod est etiam impossibile.

Vel posito . . . $e < b.$ hoc est $e = b-d.$

erit . . .
$$\begin{aligned}&+bbb\\&-3.bbd\\&+3.bdd-3.bbb\\&-.ddd+3.bbd = -ccc\\&\qquad\qquad\qquad -2.bbb\end{aligned}$$

Ergo separatis contradictorijs, est . . . . $\begin{aligned}&+ddd\\&-3bdd = +ccc.\end{aligned}$

Sed . . . . $+ddd-3.bdd = \left.\begin{array}{l}+d-3.b\\ dd\end{array}\right|$

Ergo . . . . . $d > 3.b.$ Ergo . . . $d > b.$

Sed cum positum sit . . . . $e = b-d.$ est $b > d.$ quod est quoque impossibile.

Est igitur æquatio illa reductitia omnino impossibilis.

PROBLEMA 2.

Æquationem $aaa+3.baa = +ccc.$ posito $a = e-b.$ ad æquationem $\begin{aligned}&eee-3.bbe = +ccc\\&\qquad\qquad -2bbb.\end{aligned}$ reducere.

Ponatur $e-b = a.$

$$\left.\begin{array}{ll}\text{Erit inde . . .} & +eee-3.bee+3.bbe-bbb = +aaa\\ \text{Et . .} & ee-2.be+bb = aa\\ \text{Ac proinde . .} & +3.bee-6.bbe+3.bbb = +3.baa\end{array}\right\} = +ccc.$$

Hinc reiectis contradictorijs & ordinatis reliquis

fit . . . $\begin{aligned}&eee-3.bbe = +ccc\\&\qquad\qquad -2.bbb.\end{aligned}$ æquatio requisita cuius

radix $e = a+b.$

Atque sic facta est æquationis propositæ ad requisitam reductio imperata.

PRO-

PROBLEMA 3.

Æquationem $aaa - 3.baa = -ccc.$ posito $a = b - e.$ ad æquationem. $eee - 3.bbe = \begin{array}{l} +ccc \\ -2.bbb. \end{array}$ vel posito $a = e + b$ ad æquationem $eee - 3.bbe = \begin{array}{l} -ccc \\ +2.bbb. \end{array}$ reducere.

Ponatur primo . . . . $b - e = a.$

$$\left.\begin{array}{lll} \text{Erit inde} & -eee + 3.bee - 3.bbe + bbb = & +aaa \\ \text{Et} & ee - 2.be + bb = & aa. \\ \text{Ac proinde} & -3.bee + 6.bbe - 3.bbb = & -3.baa \end{array}\right\} = -ccc$$

Hinc reiectis contradictorijs & ordinatis reliquis fit . . . . $eee - 3.bbe = \begin{array}{l} +ccc \\ -2.bbb. \end{array}$ æquatio prima requisita cuius radix $e = b - a.$

Ponatur secundo . . . . $e + b = a.$

$$\left.\begin{array}{lll} \text{Erit inde} & eee + 3.bee + 3.bbe + bbb = & +aaa \\ \text{Et} & +ee + 2.be + bb = & aa \\ \text{Ac proinde} & -3.bee - 6.bbe - 3.bbb = & -3.baa \end{array}\right\} = -ccc.$$

Hinc reiectis contradictorijs & ordinatis reliquis, fit . . . . . . $eee - 3.bbe = \begin{array}{l} -ccc \\ +2.bbb. \end{array}$ æquatio requisita secunda cuius radix $e = a - b.$

Atque sic facta est ad vtramque requisitam reductio imperata.

PROBLEMA 4.

Æquationem $aaa + 3.baa + dda = +ccc.$ posito $a = e - b.$ ad æquationem $\begin{array}{l} eee - 3.bbe \\ +.dde \end{array} = \begin{array}{l} +.ccc \\ -2.bbb \\ +.bdd. \end{array}$ reducere.

Ponatur . . . . $e - b = a.$

$$\left.\begin{array}{lll} \text{Erit inde} & eee - 3.bee + 3bbe - bbb = & +aaa \\ \text{Et} & ee - 2.be + +bb = & aa. \\ \text{Ac proinde} & +3.bee - 6.bbe + 3.bbb = & +3.baa \\ \text{Et} & +.dde - .ddb = & +.dda \end{array}\right\} = +ccc.$$

Hinc

Hinc fit . . . $\begin{aligned} eee - 3.bbe & \\ + .dde & = + .ccc \\ & \quad - 2.bbb \\ & \quad + bdd. \end{aligned}$ æquatio requisita cuius radix $e = a + b$.

Atque sic facta est reductio imperata.

PROBLEMA 5.

Æquationem $aaa - 3.baa + dda = - ccc$. posito $a = b - e$. ad æquationem $\begin{aligned} eee - 3.bbe & \\ + .dde & = + .ccc \\ & \quad - 2.bbb \\ & \quad + .bdd \end{aligned}$ vel posito $a = e + b$. ad æquationem $\begin{aligned} eee - 3.bbe & \\ + dde & = - .ccc \\ & \quad + 2.bbb \\ & \quad - .bdd. \end{aligned}$ reducere.

Ponatur primò . . . . $-e + b = a$.

Vnde : . . . . $ee - 2.be + bb = aa$.

$$\left.\begin{aligned} &\text{Et fiat inde } -eee + 3.bee - 3.bbe + .bbb = + .aaa \\ &\text{Et } . . . -3.bee + 6.bbe - 3.bbb = - 3.baa \\ &\text{Et } . . . . - .dde + .bdd = + dda \end{aligned}\right\} = - ccc$$

Hinc reiectis contradictorijs & ordinatis reliquis, & transpositis

fit . . . . . $\begin{aligned} eee - 3.bbe & \\ + .dde & = + .ccc \\ & \quad - 2.bbb \\ & \quad + .bdd \end{aligned}$ æquatio requisita prima cuius radix $e = b - a$.

Ponatur secundo . . . . $e + b = a$

Vnde . . . . $ee + 2.be + bb = aa$.

$$\left.\begin{aligned} &\text{Et fiat inde } . . \; eee + 3.bee + 3.bbe + bbb = + .aaa \\ &\text{Et } . . . . - 3.bee - 6.bbe - 3.bbb = - 3.baa \\ &\text{Et } . . . . + .dde + .bdd = + .dda \end{aligned}\right\} = - ccc.$$

Hinc reiectis contradictorijs & ordinatis reliquis,

fit . . . . $\begin{aligned} eee - 3bbe & \\ + .dde & = - .ccc \\ & \quad + 2.bbb \\ & \quad - .ddb. \end{aligned}$ æquatio requisita secunda cuius radix $e = a - b$.

Atque

Atque sic facta est propositæ æquationis ad vtramque requisitam imperata reductio.

PROBLEMA 6.

Æquationem $aaa + 3.baa - dda = +ccc.$ posito $a = e - b$

ad æquationem
$$\begin{array}{l} eee - 3.bbe \\ \quad - .dde = + .ccc \\ \qquad\qquad\quad - 2.bbb \\ \qquad\qquad\quad - .bdd \end{array}$$
reducere.

Ponatur . . . . $e - b = a.$

Vnde . . . . $ee - 2.be + bb = aa.$

$$\left.\begin{array}{lrl} \text{Fiat inde} & eee - 3.bee + 3.bbe - bbb & = + .aaa \\ \text{Et} & + 3.bee - 6.bbe + 3.bbb & = + 3.baa \\ \text{Et} & - .dde + bdd & = - .dda \end{array}\right\} = + ccc.$$

Hinc reiectis contradictorijs & ordinatis reliquis,

fit . . . . $$\begin{array}{l} eee - 3.bbe \\ \quad - .dde = + ccc \\ \qquad\qquad\quad - 2.bbb \\ \qquad\qquad\quad - .bdd. \end{array}$$ æquatio reductitia cuius radix $e = a + b.$

Atque sic facta est æquationis propositæ ad requisitam reductio imperata.

PROBLEMA 7.

Æquationem $aaa - 3.baa - dda = -ccc.$ posito $a = b - e.$

ad æquationem
$$\begin{array}{l} eee - 3.bbe \\ \quad - .dde = + .ccc \\ \qquad\qquad\quad - 2.bbb \\ \qquad\qquad\quad - .bdd. \end{array}$$
vel posito $a = b + e.$

ad æquationem
$$\begin{array}{l} eee - 3.bbe \\ \quad - .dde = - .ccc \\ \qquad\qquad\quad + 2.bbb \\ \qquad\qquad\quad + .bdd. \end{array}$$
reducere.

Ponatur . . . . $-e + b = a.$

Vnde . . . . $ee - 2.be + bb = aa.$

Fiat

$$\left.\begin{array}{ll} \text{Fiat inde} \ldots & -eee+3.bee-3.bbe+.bbb = +.aaa \\ \text{Et} \ldots & -3.bee+6.bbe-3.bbb = -3.baa \\ \text{Et} \ldots & +.dde-.bdd = -.dda \end{array}\right\} = -ccc.$$

Hinc reiectis contradictorijs & ordinatis reliquis & transpositis.

$$\text{fit} \ldots \begin{array}{l} eee-3.bbe \\ -.dde \end{array} = \begin{array}{l} +.ccc \\ -2.bbb \\ -.bda. \end{array}$$

æquatio reductitia prima cuius radix $e = +b-a$.

Ponatur secundo . . . . $e+b = a$.

Vnde. . . . . $ee++2.be+bb = aa$.

$$\left.\begin{array}{ll} \text{Fiat inde} \ldots & eee+3.bee+3.bbe+.bbb = +.aaa \\ \text{Et} \ldots & -3.bee-6.bbe-3.bbb = -3.baa \\ \text{Et} \ldots & -.dde-.bdd = -.dda \end{array}\right\} = -ccc.$$

Hinc reiectis contradictorijs & ordinatis reliquis.

$$\text{fit} \ldots \begin{array}{l} eee-3.bbe \\ -.dde \end{array} = \begin{array}{l} -ccc \\ +2.bbb \\ +.bdd. \end{array}$$

æquatio reductitia secunda cuius radix $e = a-b$.

Atque sic facta est propositæ æquationis ad requisitas imperata reductio.

PROBLEMA 8.

Æquationem $aaa-3.baa-dda = +ccc$. posito $a = e+b$ ad æquationem

$$\begin{array}{l} eee-3\,bbe \\ -.dde \end{array} = \begin{array}{l} +ccc \\ +2\,bbb \\ +.bdd. \end{array}$$

vel posito $a = b-e$. ad æquationem

$$\begin{array}{l} eee-3.bbe \\ -.dde \end{array} = \begin{array}{l} +.ccc \\ -2\,bbb \\ -.bdd. \end{array}$$

reducere.

Ponatur primo . . . . . $e+b = a$.

Vnde . . . . $ee+2.be+bb = aa$.

$$\left.\begin{array}{ll} \text{Fiat inde} \ldots & eee+3.bee+3.bbe+bbb = +.aaa \\ \text{Et} \ldots & -3.bee-6.bbe-3.bbb = -3.baa \\ \text{Et} \ldots & -.dde-.bdd = -dda \end{array}\right\} = +ccc.$$

Hinc reiectis contradictorijs & ordinatis reliquis.

fit . . . . $\begin{aligned} eee - 3.bbe & \\ - .dde &= +.ccc \\ &+2\,bbb \\ &+.bdd. \end{aligned}$ æquatio reductitia prima cuius radix $e = a - b$.

Ponatur secundo . . . . $-e + b = a$.

Vnde . . . . $ee - 2.be + bb = aa$.

$$\left.\begin{array}{ll} \text{Fiat deinde} & -eee + 3.bee - 3.bbe + .bbb = +.aaa \\ \text{Et} \ldots & -3.bee + 6.bbe - 3.bbb = -3.baa \\ \text{Et} \ldots & +..dde - ..bdd = -..dda \end{array}\right\} = +ccc.$$

Hinc reiectis contradictorijs & ordinatis reliquis & transpositis.

fit . . . . $\begin{aligned} eee - 3.bbe & \\ - ..dde &= -ccc \\ &-2\,bbb \\ &-bdd. \end{aligned}$ æquatio reductitia secunda cuius radix $e = b - a$.

Atque sic facta est propositæ æquationis ad requisitas imperata reductio.

PROBLEMA 9.

Æquationem $aaa + 3.baa - dda = -ccc.$ posito $a = -e - b$ ad æquationem $\begin{aligned} eee - 3.bbe & \\ - ..dde &= +..ccc \\ &+2.\,bbb \\ &+..bdd. \end{aligned}$ vel posito $a = e - b$

ad æquationem $\begin{aligned} eee - 3.bbe & \\ - ..dde &= -ccc \\ &-2\,bbb \\ &-bdd. \end{aligned}$ reducere.

Ponatur primo . . . . $+e - b = a$.

Vnde . . . . $ee + 2.be + bb = aa$.

$$\left.\begin{array}{ll} \text{Fiat inde} \ldots & -eee - 3.bee - 3.bbe - bbb = +aaa \\ \text{Et} \ldots & +3.bee + 6.bbe + 3.bbb = +3.baa \\ \text{Et} \ldots & +..dde + bdd = -.dda \end{array}\right\} = -ccc$$

Hinc reiectis contradictorijs, ordinatis & transpositis reliquis,

fit . . . . $\begin{aligned} eee - 3.bbe & \\ - ..dde &= +..ccc \\ &+2.bbb \\ &+..bdd. \end{aligned}$ æquatio reductitia prima cuius radix $e = -a - b$.

Ponatur

Ponatur secundo . . . . $e - b = a$.

Vnde . . . . $ee - 2.be + bb = aa$.

Fiat inde . . . $eee - 3.bee + 3.bbe - bbb = +.aaa$

Et . . . . $+ 3.bee - 6.bbe + 3.bbb = + 3.baa$ $\Big\} = -ccc$.

Et . . . . $-.dde + .bdd = -.dda$

Hinc reiectis contradictorijs & ordinatis reliquis,

fit . . . . $eee - 3\,bbe$
$-.dde = -.ccc$
$-2.bbb$
$-.bdd$. æquatio requisita secunda cuius radix $e = a + b$.

Atque sic facta est propositæ æquationis ad requisitas imperata reductio.

PROBLEMA 10.

Æquationem $aaa - 3\,baa + dda = + ccc$. posito $a = e + b$. ad æquationem $eee - 3.bbe$
$+ .dde = + .ccc$
$+ 2.bbb$
$-.bdd$ vel posito $a = -e + b$. ad æquationem $eee - 3.bbe$
$+ dde = -.ccc$
$-2.bbb$
$+.bdd$. reducere.

Ponatur primo . . . . $e + b = a$.

Vnde . . . . $ee + 2.be + bb = aa$.

Fiat inde . . . $eee + 3.bee + 3.bbe + ..bbb = + ..aaa$

Et . . . $-3.bee - 6.bbe - 3.bbb = -3.baa$ $\Big\} = + ccc$.

Et . . . . $+.dde + ..bdd = + ..dda$

Hinc reiectis contradictorijs & ordinatis reliquis

fit . . . . . $eee - 3\,bbe$
$+.dde = +.ccc$
$+2.bbb$
$-.ddb$ æquatio reductitia cuius radix $e = a - b$.

Ponatur secundo . . . . $-e + b = a$.

Vnde . . . . $ee - 2.be + bb = aa$.

Fiat

$$\left.\begin{array}{lll}\text{Fiat inde . . .} & eee+3.bee-3.bbe+..bbb & =+..aaa\\ \text{Et . . .} & -3.bee+6.bbe-3.bbb & =-3baa\\ \text{Et . . . .} & -..dde+..bdd & =+..dda\end{array}\right\}=+ccc.$$

Hinc reiectis contradictorijs & ordinatis & transpositis reliquis,

fit . . . . . . $\begin{array}{l}eee-3.bbe\\ +..dde\end{array}=\begin{array}{l}-..ccc\\ -2bbb\\ +..bdd.\end{array}$ æquatio reductitia secunda cuius radix $a=b-e$.

Atque sic facta est propositæ æquationis ad requisitas imperata reductio.

PROBLEMA II.

Æquationem $aaa+baa+dda=-ccc$. posito $a=-e-b$.

ad æquationem $\begin{array}{l}eee-3.bee\\ +..ddd\end{array}=\begin{array}{l}+..ccc\\ +2.bbb\\ -..bdd.\end{array}$ vel posito $a=e-b$.

ad æquationem $\begin{array}{l}eee-3.bbe\\ +.dde\end{array}=\begin{array}{l}-..ccc\\ -2.bbb\\ +..bdd.\end{array}$ reducere.

Ponatur primò . . . . . $-e-b=a$.

Vnde . . . . $ee+2.be+bb=aa$.

$$\left.\begin{array}{lll}\text{Fiat deinde} & -eee-3.bee-3.bbe-.bbb & =+.aaa\\ \text{Et . . .} & +3.bee+6.bbe+3.bbb & =+3.baa\\ \text{Et . . . .} & -.dde-.bdd & =+dda\end{array}\right\}=-ccc$$

Hinc reiectis contradictorijs & transpositis reliquis,

fit . . . . . . $\begin{array}{l}eee-3.bbe\\ +..dde\end{array}=\begin{array}{l}+..ccc\\ +2.bbb\\ -..bdd.\end{array}$ æquatio reductitia prima cuius radix $e=-a-b$.

Ponatur secundò . . . . . $e-b=a$.

Vnde . . . . $ee-2.be+bb=aa$.

$$\left.\begin{array}{lll}\text{Fiat inde . . .} & eee-bee+3.bbe-..bbb & =+..aaa\\ \text{Et . . .} & +3.bee-6.bbe+3.bbb & =+3.baa\\ \text{Et . . . . . .} & +..dde-..bdd & =+..dda\end{array}\right\}=-ccc.$$

Hinc reiectis contradictorijs & ordinatis reliquis,

fit . . . . . . $\begin{array}{l} aaa-3.bba \\ +..dda \end{array} = \begin{array}{l} -..ccc \\ -2.bbb \\ +.bdd. \end{array}$ æquatio reductitia secunda cuius radix $e = a+b$.

Atque sic facta est propositæ æquationis ad requisitas imperata reductio.

PROBLEMA 12.

Æquationem $aaa+3.bba = +2.ccc.$ posito $a = \frac{ee-bb}{e}$ ad æquationem simplicem $eee = +ccc+\sqrt{cccccc+bbbbbb}$. reducere.

Ponantur tres continuè proportionales $e.\ b.\ \frac{bb}{e}.$

Et sit propositæ æquationis radix supposititia a. extremarum differentiæ æqualis, videlicet $a = e-\frac{bb}{e}$. hoc est $a = \frac{ee-bb}{e}$.

$$\left.\begin{array}{ll} \text{Fiat inde } \dfrac{\begin{array}{c|c|c|c} +ee & -3.bb & +3.bb & -bb \\ ee & ee & bb & bb \\ ee & ee & ee & bb \end{array}}{eee} & = +..aaa \\ \\ \text{Et } \ldots\ \dfrac{\begin{array}{c|c} +3.bb & -3.bb \\ ee & bb \\ ee & ee \end{array}}{eee} & = +3.bba \end{array}\right\} = 2.ccc.$$

Ergo reiectis contradictorijs, & ordinatis reliquis,

Erit . . . $+eeeeee-bbbbbb = +2\,ccceee.$

Ergo . . . . $+eeeeee-2.ccceee = +\ bbbbbb.$

Ergo . . . . $+eeeeee-2.ccceee+cccccc = +cccccc+bbbbbb.$

Ergo . . . . $eee = ccc+\sqrt{cccccc+bbbbbb}.$ Quæ est æquatio præscripta.

Sic igitur facta est æquationis propositæ ad præscriptam reductio imperata.

Consectarium.

Ex reductione ista propositæ æquationis $aaa+3.bba = +2.ccc.$ ipsius resolutio opportunè deducitur.

Nam

Nam sunt $\overbrace{ccc + \surd cccccc + bbbbbb}^{I}$. $\overbrace{bbb}^{II}$ $\overbrace{-ccc + \surd cccccc + bbbbbb}^{III}$.
continuè proportionales.

Ergo etiam $\overbrace{\surd ccc + \surd cccccc + bbbbbb}^{I}$. $\overbrace{b}^{II}$. $\overbrace{\surd ccc + \surd - cccccc + bbbbbb}^{III}$.

Sed . . . . $\surd ccc + \surd cccccc + bbbbbb = e.$

Ergo . . . $\surd ccc + \surd - cccccc + bbbbbb = \frac{bb}{e}$

Ergo . . . . $\surd ccc + \surd cccccc + bbbbbb - - \surd ccc + \surd cccccc + bbbbbb = a.$

Est igitur radicalis ista binomia binomijs radicalibus implicata, radix propositæ æquationis explicatoria, quæ exhibenda erat.

Exempla resolutionis in numeris.

$20 = 6.a + aaa$ . . $a = \surd 3.)\surd 108 + 10 - \surd 3.)\surd 108 - 10. = 2.$

$26 = 9.a + aaa$ . . $a = \surd 3)\surd 196 + 13 - \surd 3)\surd 196 - 13 = 2.$

$7 = 6.a + aaa$ . . $a = \surd 3.)\surd \frac{81}{4} + \frac{7}{2} - \surd 3.)\surd 3.\frac{81}{4} - \frac{7}{2} = 1.$

PROBLEMA 13.

Æquationem $aaa - 3.bba = +2.ccc.$ posito $a = \frac{ee+bb.}{e}$ si $c.$ maior sit quam $b.$ ad æquationem simplicem. $eee = ccc + \ldots ddd.$ si $c = b.$ ad æquationem item simplicem $eee = ccc.$ Si verò $c.$ minor sit quam $b.$ ad æquationem $eee = ccc + \surd - dddddd.$ impossibilem reducere.

Ponantur tres continuè proportionales $e.$ $b.$ $\frac{bb}{e.}$

Et sit propositæ æquationis radix $a.$ extremarum summæ æqualis, videlicet $a = e + \frac{bb}{e.}$ hoc est $a = \frac{ee+bb}{e.}$

$$\left.\begin{array}{l} \text{Fiat inde } \dfrac{+eeeeee + 3.bbeeee + 3.bbbbee + bbbbbb}{eee} = +aaa \\[2ex] \text{Et} \ldots\ldots \dfrac{-3.bbeeee + 3.bbbbee}{eee.} = +3.bba \end{array}\right\} = +2.ccc.$$

Ergo

Ergo reiectis contradictorijs, & ordinatis reliquis,
fit . . . $eeeeee + bbbbbb = + 2.ccceee.$

Ergo . . . . $eeeeee - 2.ccceee = - bbbbbb.$

Ergo . . . . $eeeeee - 2.ccceee + cccccc = + cccccc - bbbbbb.$

Ergo . . . . . $eee = ccc + \sqrt{cccccc - bbbbbb}.$

Si iam ex primi casus hypothesi $c.$ maior sit quam $b.$ ponatur
$cccccc - bbbbbb = dddddd.$

Erit inde . . . $eee = ccc + \sqrt{dddddd}.$

Hoc est . . . . $eee = ccc + ddd.$ quæ est primi casus æquatio præscripta.

Si autem ex secundi casus hypothesi $b.$ æqualis sit ipsi $c.$ erit
$cccccc - bbbbbb = 0.$

Erit igitur inde . . . $eee = ccc.$ secundi casus æquatio præscripta.

Si vero ex tertij casus hypothesi $c.$ maior sit quam $b.$ ponatur
$cccccc - bbbbbb = - dddddd.$

Erit inde . . . . $eee = ccc + \sqrt{-dddddd}.$ tertij casus æquatio præscripta (propter $\sqrt{-dddddd}.$ inexplicabilitatem) impossibilis.

Sic igitur factæ sunt æquationis propositæ ad præscriptas reductiones imperatæ.

Consectarium primi casus.

Ex reductione ista propositæ æquationis $aaa - 3.bba = + 2.ccc.$ ipsius resolutio quoque deducitur.

Nam sunt $\overbrace{ccc + \sqrt{cccccc - bbbbbb}}^{I.}$ $\overbrace{bbb}^{II.}$ $\overbrace{ccc - \sqrt{cccccc - bbbbbb}}^{III.}$ continuè proportionales.

Ergo etiam $\overbrace{\sqrt{ccc + \sqrt{cccccc - bbbbbb}}}^{I.}$ $\overbrace{b}^{II.}$ $\overbrace{\sqrt{ccc - \sqrt{cccccc - bbbbbb}}}^{III.}$

Sed . . . . $\sqrt{ccc + \sqrt{cccccc - bbbbbb}} = e.$

Ergo . . . . $\sqrt{ccc - \sqrt{cccccc - bbbbbb}} = \frac{bb}{e}.$

Ergo . . . $\sqrt{ccc + \sqrt{cccccc - bbbbbb}} + \sqrt{ccc - \sqrt{cccccc - bbbbbb}} = a.$

Sit igitur radicalis ista binomia binomijs radicalibus implicata, radix propositæ æquationis explicatoria, quæ exhibenda erat.

Exempla resolutionis in numeris.

$40 = - 6.a + aaa$ . . . $a = \sqrt{3.)\ 20. + \sqrt{392.}} + \sqrt{3.)\ 20. - \sqrt{392}} = 4.$

$27 = - 24\,a + aaa$ . . $a = \sqrt{3.)\ 36. + \sqrt{784}} + \sqrt{3.)\ 36. - \sqrt{784}} = 6.$

$9 = -6.a + aaa$. ⌐ . $a = \sqrt[3]{\frac{9}{2}+\sqrt{\frac{81}{4}}} + \sqrt[3]{\frac{9}{2}-\sqrt{\frac{81}{4}}} = 3.$

Nota I.

Æquationem istam $aaa - 3.bba = +2.ccc.$ propter similitudinem quæ inter tres illius casus, & sectiones conicas hyberbolem, parabolam, & ellipsim, in triplici differentia excessus, æqualitatis & defectus intercedit, similibus nominibus, hyberbolicam scilicet, parabolicam & ellipticam appellare licet. Hyperbolicam in qua $c.$ maior est quam $b.$ parabolicam in qua $c.$ ipsi $b.$ æqualis est, ellipticam in qua $c.$ minor est quam $b.$ atque eam ob causam (in specie) irresolubilem.

Nota 2.

In duabus antecedentibus æquationibus accidit interdum binomia cubica solutionis radicalibus implicata explicari posse per radices itidem binomias, quæ per summam vel differentiam constituant tandem radicem simplicem æquationis explicatoriam. Huius generis solutionum exempla sunt quæ sequuntur.

$52 = -3.a + aaa$ . . . $a = 4.$

$$a = \underbrace{\underbrace{\sqrt[3]{26+\sqrt{675}}}_{2.+\sqrt{3}} + \underbrace{\sqrt[3]{26-\sqrt{675}}}_{2-\sqrt{3}}}_{4.}$$

$270 = +9a + aaa$ . . . $a = 6.$

$$a = \underbrace{\underbrace{\sqrt[3]{\sqrt{18252}+135}}_{\sqrt{12}+3} - \underbrace{\sqrt[3]{\sqrt{18252}-135}}_{\sqrt{12}-3}}_{6.}$$

$40 = -6.a + aaa$ . . . $a = 4.$

$$a = \underbrace{\underbrace{\sqrt[3]{20+\sqrt{392}}}_{2.+\sqrt{2}.} + \underbrace{\sqrt[3]{20-\sqrt{292}}}_{2-\sqrt{2}.}}_{4.}$$

$20 = +6a + aaa$ . . . $a = 2.$

$$a = \underbrace{\underbrace{\sqrt[3]{\sqrt{108}+10}}_{\sqrt{3}+1} - \underbrace{\sqrt[3]{\sqrt{108}-10}}_{\sqrt{3}-1.}}_{2.}$$

$$\sqrt{.21632} = -6.a + aaa \quad \ldots \quad a = \sqrt{.32}.$$

$$a = \underbrace{\underbrace{\sqrt{3.)\,5408 + \sqrt{.5400}}}_{\sqrt{.8} + \sqrt{.6}} + \underbrace{\sqrt{3)\,\sqrt{.5408} - \sqrt{.5400}}}_{\sqrt{.8} - \sqrt{.6}}}_{\sqrt{.32}}.$$

$$\sqrt{248832} = +24.a + aaa \quad \ldots \quad a = \sqrt{.48}.$$

$$a = \underbrace{\underbrace{\sqrt{3)\,\sqrt{.62720} + \sqrt{62208}}}_{\sqrt{20} + \sqrt{.12}} - \underbrace{\sqrt{3)\,\sqrt{.62720} - \sqrt{62208}}}_{\sqrt{.20} - \sqrt{.12}}}_{\sqrt{.48}}.$$

PROBLEMA 14.

Æquationem $aaaa + 4.baaa = +cccc.$ posito $a = e - b$ ad æquationem $eeee - 6.bbee + 8.bbbe = +.cccc - 3.bbbb.$ reducere.

Ponatur . . . . . . $e - b = a.$

Vnde . . . . . . $eee - 3.bee + 3.bbe - bbb = aaa.$

$$\left.\begin{array}{l} \text{Et fiat} \;..\; eeee - 4.beee + 6.bbee - 4.bbbe + bbbb = +aaaa \\ \text{Et} \;...\; +4.beee - 12.bbee + 12.bbbe - 4.bbbb = +4.baaa \end{array}\right\} = +cccc.$$

Hinc reiectis contradictorijs & ordinatis reliquis,

fit . . . . . $eeee - 6.bbee + 8.bbbe = +..cccc + 3.bbbb.$ æquatio requisita, cuius radix $e = a + b.$

Atque sic facta est imperata reductio.

PROBLEMA 15.

Æquationem $aaaa + 4.baaa = +cccc.$ posito $a = e + b.$ ad æquationem $eeee - 6.bbee - 8.bbbe = +cccc. +3.bbbb$ vel posito $a = -e + b.$ ad æquationem $eeee - 6.bbee + 8.bbbe = +cccc + 3.bbbb.$ reducere.

Ponatur primò . . . . $e + b = a.$

Vnde

Vnde . . . . $eee + 3.bee + 3.bbe + bbb = aaa$.

$$\left.\begin{array}{ll}\text{Et fiat . .} & eeee + 4.beee + 6.bbee + 4\,bbbe + bbbb = + .. aaaa \\ \text{Et . . . . .} & -4.beee - 12.bbee - 12.bbbe - 4.bbbb = -4.baaa\end{array}\right\} = + cccc.$$

Hinc reiectis contradictorijs & ordinatis reliquis,
fit . . . . $eeee - 6.bbee - 8.bbbe = + .. cccc + 3.bbbb$. æquatio prima, cuius radix $e = a - b$.

Ponatur secundò . . . . . . $-e + b = a$.

Vnde . . . . : . $-eee + 3.bee - 3.bbe + bbb = aaa$.

$$\left.\begin{array}{ll}\text{Et fiat . .} & eeee - 4\,beee + 6.bbee - 4.bbbe + bbbb = + .aaaa \\ \text{Et . . .} & + 4\,beee - 12.bbee + 12.bbbe - 4\,bbbb = -4.baaa\end{array}\right\} = + cccc.$$

Hinc reiectis contradictorijs & ordinatis reliquis,
fit . . . . . . $eeee - 6.bbee + 8.bbbe = + cccc + 3\,bbbb$. æquatio requisita secunda, cuius radix $e = + b - a$.

Atque sic facta est propositæ æquationis ad requisitas imperata reductio.

PROBLEMA 16.

Æquationem $aaaa - 4.baaa = -cccc$. posito $a = e + b$. ad æquationem $eeee - 6\,bbee - 8.bbbe = - .. cccc + 3.bbbb$. vel posito $a = -e + b$. ad æquationem $eeee - 6.bbee + 8.bbbe = -cccc + 3.bbbb$. reducere.

Ponatur primò : . . . . . $e + b = a$.

Vnde : . . . . $eee + 3.bee + bbe + bbb = aaa$.

$$\left.\begin{array}{ll}\text{Et fiat . .} & eeee + 4.beee + 6.bbee + 4\,bbbe + bbbb = + .aaaa \\ \text{Et . . . .} & -4.beee - 12.bbee - 12.bbbe - 4.bbbb = -4.baaa\end{array}\right\} = -cccc.$$

Hinc reiectis contradictorijs & ordinatis reliquis,
fit . . . . $eeee - 6\,bbee - 8\,bbbe = - .. cccc + 3.bbbb$. æquatio requisita prima, cuius radix $e = a - b$.

Ponatur secundò . . . . $-e + b = a$.

Vnde . . . . $-eee + 3.bee - 3.bbe + bbb = aaa$.

Et

Et fiat . . $eeee - 4\,beee + 6\,bbee - 4.bbbe + \,..\,bbbb = + \,..\,aaaa$
Et . . . . $+4\,beee - 12.bbee + 12.bbbe - 4.bbbb = -\,4.baaa$ $\Big\} = -cccc$

Hinc reiectis contradictorijs & ordinatis reliquis,
fit . . . . $eeee - 6.bbee + 8.bbbe = -\,..\,cccc$
$+\,3.bbbb.$ æquatio requisita secunda, cuius radix $e = b - a.$

Atque sic facta est propositæ æquationis ad requisitas imperata reductio.

PROBLEMA 17.

Æquationem $aaaa + 4.baaa = -cccc.$ posito $a = e - b.$
ad æquationem $eeee - 6.bbee + 8.bbbe = -cccc$
$+3.bbbb.$ reducere.

Ponatur . . . . $e - b = a.$

Vnde . . . . $eee - 3.bee + 3.bbe - bbb = aaa.$

Et fiat . . $eeee - 4.beee + 6.bbee - 4.bbbe + \,..\,bbbb = +\,4\,aaaa$
Et . . . $+\,4.beee - 12.bbee - 12.bbbe + \,..\,bbbb = +\,4.baaa$ $\Big\} = -cccc.$

Hinc reiectis contradictorijs & ordinatis reliquis,
fit . . . . $eeee - 6.bbee + 8.bbbe = -cccc$
$+\,3.bbbb.$ æquatio requisita. cuius radix $e = a + b.$

Atque sic facta est imperata reductio.

PROBLEMA 18.

Æquationem $aaaa + 4.baaa + ddda = +cccc.$ posito $a = e - b.$
ad æquationem $eeee - 6.bbee + 8\,bbbe$
$+\,..\,ddde = +cccc$
$+\,3.bbbb$
$+\,..\,bddd.$ reducere.

Ponatur . . . . $e - b = a.$

Vnde . . . . . . $eee - 3.bee + 3.bbe - bbb = aaa.$

Et fiat . . $eeee - 4.beee + 6.bbee - 4.bbbe + \,..\,bbbb = +\,..\,aaaa$
Et . . . . $+\,4\,beeee - 12.bbee - 12.bbbe - 4.bbbb = +\,4.baaa$
Et . . . . $+\,..\,ddde - \,..\,bddd = +\,.\,ddda$ $\Big\} = +cccc.$

Hinc reiectis contradictorijs & ordinatis reliquis,

fit

fit . . . . . .

$$\begin{array}{l} eeee - 6.bbee + 8.bbbe \\ + ..ddde = + cccc \\ \qquad\qquad + 3.bbbb \\ \qquad\qquad + bddd. \end{array}$$

æquatio requisita, cuius radix $e = a + b$.

Atque sic facta est imparata reductio.

PROBLEMA 19.

Æquationem $aaaa - 4.baaa - ddda = + cccc.$ posito $a = - e + b.$ ad æquationem

$$\begin{array}{l} eeee - 6.bbee + 8bbbe \\ + .ddde = + cccc \\ \qquad\qquad + 3.bbbb \\ \qquad\qquad + bddd \end{array}$$

vel $a = e + b.$ ad equationem

$$\begin{array}{l} eeee - 6.bbee - 8.bbbe \\ - ..ddde = + cccc \\ \qquad\qquad + 3.bbbb \\ \qquad\qquad + .bddd. \end{array}$$

reducere.

Ponatur primò . . . . . . $- e + b = a.$

Vnde . . . . . . $- eee + 3.bee - 3.bbe + bbb = aaa.$

$$\left.\begin{array}{ll} \text{Et fiat} .. & eeee - 4.beee + 6.bbee - 4.bbbe + ..bbbb = + ..aaaa \\ \text{Et} . . . . & + 4.beee - 12.bbee + 12.bbbe - 4.bbbb = - 4.baaa \\ \text{Et} . . . . & + ..ddde - .bddd = - .ddda \end{array}\right\} = + cccc.$$

Hinc reiectis contradictorijs & ordinatis reliquis,

fit . . . . . .

$$\begin{array}{l} eeee - 6bbee + 8bbbe \\ + ..ddde = - ..cccc \\ \qquad\qquad + 3bbbb \\ \qquad\qquad + bddd. \end{array}$$

æquatio requisita prima, cuius radix $e = - a + b.$

Ponatur secundò . . . . $e + b = a.$

Vnde . . . . $+ eee + 3.bee + 3.bbe + bbb = aaa.$

$$\left.\begin{array}{ll} \text{Et fiat} .. & eeee + 4.beee + 6.bbee + 4.bbbe + ..bbbb = + ..aaaa \\ \text{Et} . . . . & - 4beee - 12.bbee - 12.bbbe - 4.bbbb = - 4baaa \\ \text{Et} . . . . & - ..ddde - ..bddd = - .ddda \end{array}\right\} = + cccc.$$

Hinc reiectis contradictorijs & ordinatis reliquis,

fit . . . . . .

$$\begin{array}{l} eeee - 6.bbee - 8.bbbe \\ - ..ddde = + ..cccc \\ \qquad\qquad + 3.bbbb \\ \qquad\qquad + ..bddd. \end{array}$$

æquatio

tio requisita secunda, cuius radix $e = a - b$.

Atque sic facta est propositæ æquationis ad requisitas imperata reductio.

PROBLEMA 20.

Æquationem $aaaa + 4.baaa + ffaa = + cccc$. posito $a = e - b$. ad æquationem

$$\begin{array}{l} eeee - 6.bbee + 8.bbbe \\ \quad + ..ffee - 2.bffe = + ..cccc \\ \qquad\qquad\qquad\qquad + 3.bbbb. \\ \qquad\qquad\qquad\qquad - .bbff. \end{array}$$

reducere.

Ponatur . . . . . . $e - b = a$.

Vnde . . . . . . $ee - 2.be + be - bb = aa$.

Et . . . . $eee - 3.bee + 3.bbe - bbb = aaa$.

$$\left.\begin{array}{ll} \text{Et fiat} & eeee - 4.beee + 6.bbee - 4\,bbbe + bbbb = + aaaa \\ \text{Et} & + 4.beee - 12.bbee + 12.bbbe - 4.bbbb = + 4.baaa \\ \text{Et} & + ..ffee - 2\,bffe + ..bbff = + ..ffaa \end{array}\right\} = + cccc.$$

Hinc reiectis contradictorijs & ordinatis reliquis,

fit . . . .
$$\begin{array}{l} eeee - 6.bbee + 8\,bbbe \\ \quad + ..ffee - 2.bffe = + ..cccc \\ \qquad\qquad\qquad\qquad + 3.bbbb \\ \qquad\qquad\qquad\qquad - ..bbff. \end{array}$$
æquatio requisita, cuius radix $e = a + b$.

Atque sic facta est imperata reductio.

PROBLEMA 21.

Æquationem $aaaa - 4.baaa + ffaa = + cccc$. posito $a = - e + b$. ad æquationem

$$\begin{array}{l} eeee - 6.bbee + 8.bbbe \\ \quad + .ffee - 2.bffe = + cccc \\ \qquad\qquad\qquad\qquad + 3\,bbbb \\ \qquad\qquad\qquad\qquad - .bbff. \end{array}$$

vel posito $a = + e + b$. ad æquationem

$$\begin{array}{l} eeee - 6.bbee - 8.bbbe \\ \quad + .ffee + 2.bbff = + cccc \\ \qquad\qquad\qquad\qquad + 3.bbbb \\ \qquad\qquad\qquad\qquad - .bbff. \end{array}$$

reducere.

Ponatur primò . . . . $- e + b = a$.

Vnde

Vnde . . . . $ee - 2.be + bb = aa$.

Et . . . . . . $-eee + 3.bee - 3.bbe + bbb = aaa$.

$$\left.\begin{array}{ll} \text{Et fiat . .} & eeee - 4.beee + 6.bbee - 4.bbbe + .bbbb = +.aaaa \\ \text{Et . . . . .} & +4.beee - 12.bbee + 12.bbbe - 4.bbbb = -4.baaa \\ \text{Et . . . .} & +..ffee - 2.bffe + ..bbff = +..ffaa \end{array}\right\} = +cccc.$$

Hinc reiectis contradictorijs & ordinatis reliquis,

fit . . . . . $\begin{array}{l} eeee - 6.bbee + 8.bbbe \\ +..ffee - 2.bffe = \begin{array}{l} +..cccc \\ +3.bbbb \\ -3.bbff. \end{array} \end{array}$ æquatio requisita prima, cuius radix $e = -a + b$.

Ponatur secundò . . . . . $e + b = a$.

Vnde . . . . $ee + 2.be + bb = aa$.

Et . . . . $eee + 3.bee + 3bbe + bbb = aaa$.

$$\left.\begin{array}{ll} \text{Et fiat . .} & eeee + 4.beee + 6.bbee + 4.bbbe + bbbb = +.aaaa \\ \text{Et . . .} & -4.beee - 12.bbee - 12.bbbe - 4\,bbbb = -4.baaa \\ \text{Et . . . . . .} & +.ffee + 2.bffe + ..bbff = +.ffaa \end{array}\right\} = +cccc.$$

Hinc reiectis contradictorijs & ordinatis reliquis,

fit . . . . . . $\begin{array}{l} eeee - 6.bbee - 8.bbbe \\ +..ffee + 2.bffe = \begin{array}{l} +.cccc \\ +3\,bbbb \\ -b.bff. \end{array} \end{array}$ æquatio requisita secunda, cuius radix $e = +a - b$.

Atque sic facta est propositæ æquationis ad requisitas imperata reductio.

PROBLEMA 22.

Æquationem $aaaa + 4.baaa + ffaa + ddda = +cccc$. posito $a = e - b$. ad æquationem

$$\begin{array}{l} eeee - 6.bbee + 8.bbbe \\ \quad +..ffee - 2.bffe \\ \quad\quad +..ddde = \begin{array}{l} +cccc \\ +3.bbbb \\ -.bbff \\ +.bddd. \end{array} \end{array}$$

reducere.

Ponatur . . . . $e - b = a$.

Vnde . . . . $ee - 2.be + bb = aa$.

Et . . . . $eee - 3.bee + 3.bbe - bbb = aaa$.

Et

$$\left.\begin{array}{llr}\text{Et fiat} & eeee - 4.beee + 6.bbee - 4bbbe + bbbb & = +.aaaa \\ \text{Et} & +4.beee - 12.bbee + 12.bbbe - 4.bbbb & = +.4.baaa \\ \text{Et} & +..ffee - .2.bffe + .bbff & = +..ffaa \\ \text{Et} & +..ddde - ..bddd & = +..ddda\end{array}\right\} = +cccc.$$

Hinc reiectis contradictorijs & ordinatis reliquis,

fit $eeee - 6.bbee + 8.bbbe + ..ffee - 2.bffe + .ddde = +.cccc + 3.bbbb - .bbff + .bddd.$ æquatio requisita, cuius radix $e = a + b.$

Atque sic facta est imperata reductio.

PROBLEMA 23.

Æquationem $aaaa - 4.baaa + ffaa - ddda = + cccc.$ posito $a = - e + b.$ ad æquationem $eeee - 6.bbee + 8.bbbe + ..ffee - 2.bffe + .ddde = + cccc + 3.bbbb - .bbff + bddd.$ vel posito $a = + e + b.$ ad æquationem $eeee - 6.bbbe - 8.bbbe + ..ffee + 2.bffe - .ddde = +.cccc + 3.bbbb - .bbff + ..bddd$ reducere.

Ponatur primò $- e + b = a.$

Vnde $ee - 2.be + bb = aa.$

Et $- eee + 3.bee + bbe + bbb = aaa.$

$$\left.\begin{array}{llr}\text{Et fiat} & eeee - 4.beee + 6.bbee - 4.bbbe + ..bbbb & = +4.aaaa \\ \text{Et} & +4.beee - 12.bbee + 12.bbbe - 4.bbbb & = -4.baaa \\ \text{Et} & +..ffee - 2.bffe + ..bbff & = +..ffaa \\ \text{Et} & +.ddde - ..bddd & = -.ddda\end{array}\right\} = + cccc.$$

Hinc reiectis contradictorijs & ordinatis reliquis,

fit

fit . . . . . $eeee - 6.bbee + 8.bbbe$
$+ ..ffee - 2.bffe$
$+ ..ddde = + ..cccc$
$+ 3\,bbbb$
$- ..bbff$
$+ ..bddd.$ æqua-
tio requisita prima, cuius radix $e = -a + b.$

Ponatur secundò . . . . . $e + b = a.$

Vnde . . . . $ee + 2.be + bb = aa.$

Et . . . . $eee + 3.bee + 3\,bbe + bbb = aaa.$

$$\left.\begin{array}{lll} \text{Et fiat} \;.\;. & eeee + 4\,beee + 6.bbee + 4.bbbe + bbbb = & + .\,aaaa \\ \text{Et} \;.\;.\;. & - 4.beee - 12.bbee - 12.bbbe - 4\,bbbb = & - 4\,baaa \\ \text{Et} \;.\;.\;.\;.\;. & + .\,ffee + 2.bffe + ..bbff = & + .ffaa \\ \text{Et} \;.\;.\;.\;.\;.\;.\;.\;. & - ..ddde - bddd = & - .ddda \end{array}\right\} = + cccc.$$

Hinc reiectis contradictorijs & ordinatis reliquis,
fit . . . . . . $eeee - 6.bbee - 8.bbbe$
$+ ..ffee + 2.bffe$
$- ..ddde = + .cccc$
$+ 3.bbbb$
$- .bbff$
$+ .bddd.$ æ-
quatio requisita secunda, cuius radix $e = a - b.$

Atque sic facta est propositæ æquationis ad requisitas imperata reductio.

PROBLEMA 24.

Æquationem $aaaa - 4.baaa + ffaa - ddda = - cccc.$ posito $a = e + b.$ ad æquationem $eeee - 6.bbee - 8\,bbbe$
$+ ..ffee + 2.ffbe$
$- ..ddde = - .cccc$
$+ 3\,bbbb$
$- .ffbb$
$+ dddb.$

vel posito $a = - e + b.$ ad ęquationem $eeee - 6.bbee + 8.bbbe$
$+ ..ffee - 2.ffbe$
$+ ..ddde =$
$= - cccc$
$+ 3.bbbb$
$- .bbff$
$+ .bddd.$ reducere.

Harum reductionum processus antecedenti similis est.

PROBLEMA 25.

Æquationem $aaaa + 4.baaa - ffaa + ddda = +cccc$. ad æquationem

$$\begin{array}{llll} eeee - 6.bbee + 8.bbbe & = & +cccc \\ -..ffee + 2.bffe & & -3.bbbb \\ +.ddde & & +.ffbb \\ & & +.dddb. \end{array}$$

posito $a = +e-b$. reducere.

Ponatur . . . . $+e-b = a$.

Vnde . . . . . . $ee - 2.be + bb = aa$.

Et . . . . $eee - 3.bee + 3.bbe - bbb = aaa$.

$$\left.\begin{array}{lll} \text{Et fiat} \ldots & eeee - 4.beee + 6.bbee - 4.bbbe + bbbb & = +.aaaa \\ \text{Et} \ldots & +4.beee - 12.bbee + 12.bbbe - 4.bbbb & = +4.baaa \\ \text{Et} \ldots & -..ffee + .2.bffe - .bbff & = -..ffaa \\ \text{Et} \ldots & +..ddde - ..bddd & = +..ddda \end{array}\right\} = +cccc.$$

Hinc reiectis contradictorijs & ordinatis reliquis,

$$\begin{array}{llll} \text{fit} \ldots & eeee - 6.bbee + 8.bbbe & = & +.cccc \\ & -..ffee + 2ffbe & & +3.bbbb \\ & +.ddde & & +.ffbb \\ & & & +.dddb. \end{array}$$

æquatio præscripta, cuius radix $e = a+b$.

Sic igitur facta est æquationis propositæ ad præscriptam imperata reductio.

PROBLEMA 26.

Æquationem $aaaa + 4.baaa + ffaa - ddda = +cccc$. posito $a = +e-b$.

ad æquationem

$$\begin{array}{llll} eeee - 6.bbee + 8.bbbe & = & +cccc \\ +..ffee - 2.ffbe & & +3.bbbb \\ -.ddde & & -.ffbb \\ & & -dddb. \end{array}$$

reducere.

Ponatur . . . . . $+e-b = a$.

Vnde . . . . . . $ee - 2.be + bb = aa$.

Et . . . . $eee - 3.bee + 3.bbe - bbb = aaa$.

Et

$$\left.\begin{array}{ll} \text{Et fiat} \; .. & eeee - 4.beee + 6.bbee - 4.bbbe + ..bbbb = +.aaaa \\ \text{Et} \; ... & + 4.beee - 12.bbee + 12.bbbe - 4.bbbb = + 4.baaa \\ \text{Et} \; ...... & + ..ffee - 2.ffbe + ..ffbb = + ..ffaa \\ \text{Et} \; .... & - .ddde + ..ddab = - .ddda \end{array}\right\} = + cccc.$$

Hinc vero reiectis contradictorijs & ordinatis reliquis,

$$\begin{array}{lrl} \text{fit} \; ..... & eeee - 6 bbee + 8.bbbe & = + .cccc \\ & + .ffee - 2.ffbe & + 3.bbbb \\ & - ..ddde & - .ffbb \\ & & - .bddd. \end{array}$$

æquatio præscripta, cuius radix $e = a + b$.

Sic igitur facta est æquationis propositæ ad præscriptam reductio imparata.

PROBLEMA 27.

Æquationem $aaaa - 4.baaa + ffaa + ddda = + cccc$
posito $a = + e + b$.
ad æquationem

$$\begin{array}{rl} eeee - 6.bbee - 8.bbbe & = +.cccc \\ +..ffee + 2.ffbe & + 3.bbbb \\ +.ddde & - .ffbb \\ & - .dddb. \end{array}$$

vel posito $a = - e + b$. ad æquationem

$$\begin{array}{rl} eeee - 6.bbee + 8.bbee & \\ +..ffee - 2 ffbe & \\ - ..ddde & = + cccc \\ & + 3.bbbb \\ & - .ffbb \\ & - .dddb. \end{array}$$

reducere.

Ponatur primò . . . . . $+ e + b = a$.

Vnde . . . . . . $+ ee + 2.be + bb = aa$.

Et . . . . . $+ eee + 3.bee + 3.bbe + bbb = aaa$.

$$\left.\begin{array}{ll} \text{Et fiat} \; . & + eeee + 4.beee + 6.bbee + 4.bbbe + bbbb = + aaaa \\ \text{Et deinde} \; . & - 4.beee - 12.bbee - 12.bbbe - 4.bbbb = - 4.baaa \\ \text{Et} \; ...... & + ..ffee + 2.bffe + ..bbff = + ..ffaa \\ \text{Et.} \; .... & + ..ddde + .dddb = + .ddda \end{array}\right\} = + cccc.$$

Hinc vero reiectis contradictorijs & ordinatis reliquis,

$$\begin{array}{lrl} \text{fit} \; ..... & eeee - 6.bbee - 8 bbbe & = + ..cccc \\ & + ..ffee + 2.ffbe & + 3.bbbb \\ & + .ddde & - ..ffbb \\ & & - ..dddb. \end{array}$$

æquatio præ-

præscripta, cuius radix $e = +a - b$.

Ponatur secundò . . . . $-e + b = a$.

Hinc processu simili fit

$$\begin{array}{lcl} eeee - 6.bbee + 8\,bbbe & = & +..cccc \\ +.ffee - 2.ffbe & & +3.bbbb \\ -..ddde & & -.ffbb \\ & & -.dddb. \end{array}$$

æquatio præscripta secunda, cuius radix $e = -a + b$.

Sic igitur factæ sunt æquationis propositæ reductiones imperatæ.

PROBLEMA 28.

Æquationem $aaaa + 4.baaa - ffaa - ddaa = +cccc.$ posito $a = +e - b.$ ad æquationem

$$\begin{array}{lcl} eeee - 6.bbee + 8.bbbe & = & +cccc \\ -.ffee + 2.ffbe & & +3.bbbb \\ -ddde & & +.ffbb \\ & & -.dddb. \end{array}$$

reducere.

Ponatur . . . . $+e - b = a$.

Vnde . . . . . . $+ee - 2.be + bb = aa$.

Et . . . . $+eee - 3.bee + 3.bbe - bbb = aaa$.

$$\left.\begin{array}{ll} \text{Et fiat} . +eeee - 4.beee + 6.bbee - 4.bbbe + ..bbbb = +..aaaa \\ \text{Et deinde} . +4.beee - 12.bbee + 12.bbbe - 4.bbbb = +4.baaa \\ \text{Et} \ldots\ldots -..ffee + 2.ffbe - ffbb = -.ffaa \\ \text{Et} \ldots\ldots -..ddde + .bddd = -.ddda \end{array}\right\} = +cccc.$$

Hinc reiectis contradictorijs & ordinatis reliquis,

fit . . . . . .

$$\begin{array}{lcl} eeee - 6.bbee + 8.bbbe & = & +..cccc \\ -..ffee + ..ffbe & & +3.bbbb \\ -..ddde & & +..ffbb \\ & & -..dddb. \end{array}$$

æquatio præscripta, cuius radix $e = +a + b$.*

Sic igitur facta est æquationis propositæ ad præscriptam reductio imperata.

PROBLEMA 29.

Æquationem $aaaa - 4.baaa - ffaa + ddda = +cccc.$ posito $a = +e + b.$ ad æquationem

$$\begin{array}{lcl} eeee - 6.bbee - 8\,bbbe & = & +cccc \\ -.ffee - 2.ffbe & & +3.bbbb \\ +.ddde & & +.ffbb \\ & & -.dddb. \end{array}$$

vel

vel posito $a = -e + b$.

ad æquationem
$$\begin{array}{llll} eeee & -6\,bbee & +8.bbbe & = +..cccc \\ & -.ffee & +2.ffbe & +3.bbbb \\ & & -..ddde & +.ffbb \\ & & & -.dddb \end{array}$$
reducere.

Ponatur primò . . . . $+e+b = a$.

Vnde . . . . $+ee+2.be+bb = aa$.

Et . . . . $+eee+3.bee+3.bbe+bbb = aaa$.

$$\left.\begin{array}{ll} \text{Et fiat} & +eeee+4.beee+6.bbee+4\,bbbe+.bbbb = +.aaaa \\ \text{Et deinde} & -4\,beee-12.bbee-12.bbbe-4.bbbb = -4.baaa \\ \text{Et} & -..ffee-2.bffe-..bbff = -..ffaa \\ \text{Et} & +..ddde+..dddb = +.ddda \end{array}\right\} = +cccc.$$

Hinc reiectis contradictorijs & ordinatis reliquis,

fit . . . .
$$\begin{array}{llll} eeee & -6.bbee & -8.bbbe & = +.cccc \\ & -.ffee & -2\,ffbe & +3.bbbb \\ & & +.ddde & +..ffbb \\ & & & -.dddb. \end{array}$$
æquatio præscripta, cuius radix $e = +a-b$.

Ponatur secundò . . . . $-e+b = a$.

Hinc processu simili fit . . .
$$\begin{array}{llll} eeee & -6\,bbee & +8.\,bbbe & = +.cccc \\ & -..ffee & +2.ffbe & +3.bbbb \\ & & -.ddde & +.ffbb \\ & & & -.dddb. \end{array}$$
æquatio secunda, cuius radix $e = -a+b$.

Sic igitur facta est æquationis propositæ ad præscriptas reductio imperata.

PROBLEMA 30.

Æquationem $aaaa - 4.baaa - ffaa - ddda = +cccc$. posito $a = +e+b$. ad æquationem
$$\begin{array}{llll} eeee & -6.bbee & -8\,bbbe & = +.cccc \\ & -.ffee & -2.ffbe & +3.bbbb \\ & & -.ddde & +.ffbb \\ & & & +.dddb. \end{array}$$
vel posito $a = -e+b$. ad æquationem
$$\begin{array}{llll} eeee & -6.bbee & +8\,bbbe & = +cccc \\ & -..ffee & +2.ffbe & +3.bbbb \\ & & +.ddde & +.ffbb \\ & & & +dddb. \end{array}$$

reducere.

Ponatur primò . . . . . $+e+b = a.$

Vnde . . . . $+ee+2.be+bb = aa.$

Et . . . . . $+eee+3.bee+3.bbe+bbb = aaa.$

$$\left.\begin{array}{lrcl} \text{Et fiat . .} & eeee+4.beee+6.bbee+4.bbbe+..bbbb & = & +..aaaa \\ \text{Et deinde .} & -4beee-12.bbee-12.bbbe-4.bbbb & = & -4\,baaa \\ \text{Et . . . .} & -..ffee-2.ffbe-4.ffbb & = & -..ffaa \\ \text{Et . . . .} & -..ddde-..bddd & = & -.ddda \end{array}\right\} = +cccc.$$

Hinc reiectis contradictorijs & ordinatis reliquis,

$$\begin{array}{lrcl} \text{fit . . . . .} & eeee-6.bbee-8.bbbe & = & +.cccc \\ & -..ffee-2.ffbe & & +3.bbbb \\ & -.ddde & & +.ffbb \\ & & & +.dddb. \end{array}$$

æquatio præscripta, cuius radix $e = +a-b.$

Ponatur secundò . . . . . $a = -e+b.$

Hinc processu simili fit

$$\begin{array}{rcl} eeee-6.bbee+8.bbbe & = & +..cccc \\ -..ffee+2.ffbe & & +3.bbbb \\ +.ddde & & +..ffbb \\ & & +..dddb. \end{array}$$

æquatio præscripta, cuius radix $e = a+b.$

Sic igitur facta est æquationis propositæ ad præscriptas, reductio imparata.

PROBLEMA 31.

Æquationem $aaaa+4.baaa-ffaa = +cccc.$ posito $a = +e-b.$ ad æquationem

$$\begin{array}{rcl} eeee-6.bbee+8.bbbe & = & +cccc \\ -..ffee+2.ffbe & & +3.bbbb \\ & & +.ffbb. \end{array}$$

reducere.

Ponatur . . . . . $+e-b = a.$

Vnde . . . . $+ee-2.be+bb = aa.$

Et . . . . $+eee-3.bee+3.bbe-bbb = aaa.$

$$\left.\begin{array}{lrcl} \text{Et fiat .} & +eeee-4.beee+6.bbee-4.bbbe+.bbbb & = & +.aaaa \\ \text{Et deinde .} & +4.beee-12.bbee+12.bbbe-4.bbbb & = & +4.baaa \\ \text{Et . . . . . .} & -..ffee+2.ffbe-..ffbb & = & -..ffaa \end{array}\right\} = +cccc.$$

Hinc reiectis contradictorijs & ordinatis reliquis,

$$\begin{array}{lrcl} \text{fit . . . . .} & eeee-6\,bbee+8.bbbe & = & +.cccc \\ & -,ffee+2.ffbe & & +3.bbbb \\ & & & +.ffbb. \end{array}$$

æquatio

præ-

præscripta, cuius radix $e = a + b$.

Sic igitur facta est æquationis propositæ ad præscriptam reductio imperata.

PROBLEMA 32.

Æquationem $aaaa - 4.baaa - ffaa = +cccc$. posito $a = +e+b$.

ad æquationem
$$\begin{array}{l} eeee - 6.bbee - 8.bbbe \\ -..ffee - 2.ffbe \end{array} = \begin{array}{l} +.cccc \\ +3.bbbb \\ +.ffbb. \end{array}$$
vel posito $a = -e+b$.

ad æquationem
$$\begin{array}{l} eeee - 6.bbee + 8.bbbe \\ -..ffee + 2ffbe \end{array} = \begin{array}{l} +.cccc \\ +3.bbbb \\ +.ffbb. \end{array}$$
reducere.

Ponatur primò . . . . . $+e+b = a$.

Vnde . . . . . $+ee+2.be+bb = aa$.

Et . . . . . $+eee+3.bee+3.bbe+bbb = aaa$.

$$\begin{array}{ll} \text{Et fiat} & +eeee+4.beee+6.bbee+4.bbbe+bbbb = +aaaa \\ \text{Et deinde} & -4.beee-12.bbee-12.bbbe-4.bbbb = -4.baaa \\ \text{Et} & -..ffee-2.ffbe-..ffbb = -..ffaa \end{array}\left.\vphantom{\begin{array}{l}a\\a\\a\end{array}}\right\} = +cccc.$$

Hinc vero reiectis contradictorijs & ordinatis reliquis,

fit . . . .
$$\begin{array}{l} eee - 6.bbee - 8.bbbe \\ -..ffee - 2.ffbe \end{array} = \begin{array}{l} +..cccc \\ +3\,bbbb \\ +..ffbb. \end{array}$$
æquatio præscripta, cuius radix $e = +a-b$.

Ponatur secundò . . . . $-e+b = a$.

Hinc processu simili fit
$$\begin{array}{l} eeee - 6\,bbee + 8\,bbbe \\ -.ffee + 2\,ffbe \end{array} = \begin{array}{l} +..cccc \\ +3.bbbb \\ +.ffbb. \end{array}$$
æquatio reductitia secunda, cuius radix $e = -a+b$.

Atque sic facta est æquationis propositæ ad præscriptas reductio imperata.

PROBLEMA 33.

Æquationem $aaaa + 4.baaa - ddda = +cccc$. posito $a = +e-b$.

ad æquationem
$$\begin{array}{l} eeee - 6.bbee + 8.bbbe \\ -.ddde \end{array} = \begin{array}{l} +.cccc \\ +3.bbbb \\ -.dddb. \end{array}$$
reducere.

Ponatur

Ponatur . . . . $+e-b = a$.

Vnde . . . . . $+ee-2.be+bb = aa$.

Et . . . . $+eee-3.bee+3.bbe-bbb = aaa$.

$$\left.\begin{array}{ll}\text{Et fiat . } +eeee-4.beee+6.bbee-4.bbbe+..bbbb = & +..aaaa\\ \text{Et deinde . } +4.beee-12\,bbee+12.bbbe-4.bbbb = & +4.baaa\\ \text{Et . . . . } -..ddde+..dddb = & -.ddda\end{array}\right\} = +cccc.$$

Hinc reiectis contradictorijs & ordinatis reliquis,

fit . . . . . $\begin{array}{l}eeee-6.bbee+8.bbbe\\ \quad -..ddde\end{array} = \begin{array}{l}+..cccc\\ +3.bbbb\\ -..dddd.\end{array}$ æquatio præscripta, cuius radix $e = +a+b$.

Sic igitur facta est æquationis propositæ ad præscriptam imperata reductio.

PROBLEMA 34.

Æquationem $aaaa-4.baaa+ddda = +cccc$. posito $a = e+b$. ad æquationem $\begin{array}{l}eeee-6.bbee-8\,bbbe\\ \quad +..ddde\end{array} = \begin{array}{l}+.cccc\\ +3\,bbbb\\ -dddb.\end{array}$

reducere.

Ponatur . . . . $+e+b = a$.

Vnde . . . . $eee+3.bee+3.bbe+bbb = aaa$.

$$\left.\begin{array}{ll}\text{Et fiat . } +eeee+4\,beee+6.bbee+4.bbbe+bbbb = & +.aaaa\\ \text{Et deinde . } -4.beee-12.bbee-12.bbbe-4\,bbbb = & -4.baaa\\ \text{Et . . . . } +..ddde+dddb = & +.ddda\end{array}\right\} = +cccc.$$

Hinc vero reiectis contradictorijs & ordinatis reliquis,

fit . . . . . $\begin{array}{l}eeee-6.\,bbee-8.bbbe\\ \quad +..ddde\end{array} = \begin{array}{l}+.cccc\\ +3.bbbb\\ -.bddd.\end{array}$ æquatio præscripta, cuius radix $e = +a-b$.

Sic igitur facta est æquationis propositæ ad præscriptam reductio imperata.

Atque sic explicita est tractatus huius pars prima ad Exegesim numerosam præparatoria, secunda, quæ iam sequitur, principalis est, ipsam Exegetices numerosæ praxim continens.

EXE-

EXEGETICE NVMEROSA.

Ad æquationes quadraticas resoluendas.

PROBLEMA 1.

E Dato æquationis quadraticæ simplicis . . . $aa = ff$. in numeris propositæ homogeneo radicem radicis quæsititiæ a. valorem Analyticè reducere.

Sit æquatio numerosè proposita $aa = 48233025$.

Vnde . . . . $48233025 = ff$.

Ponatur . . . . $b+c = a$.

Ergo . . . . $\left.\begin{array}{l} b+c \\ b+c \end{array}\right\} = 48233025$.

Factis igitur & ordinatis vt oportet homogeneis particularibus,
fit . . . . . . $\underbrace{+bb}_{Ab} \quad \underbrace{\begin{array}{l} +2.bc \\ +..cc \end{array}}_{Bc} = 48233025$

Est autem æquationis huius pars speciosa bipartita Ab. Bc. resolutionis Canon cuius applicatione operis analytici processus dirigendus est, quem rite constitutum esse ex sequenti Lemmate constabit.

Factâ igitur Canonis huius applicatione omnino vt in subiecto exempli schematismo ordinata conspicitur, fiat ipsius directione dati homogenei 48233025. ad radicem ex eo educendam resolutio vt sequitur.

Radix vniuersalis successiuè educenda			6		9		4		5
Homogeneum resoluendum.	ff . . .	4	8	2	3	3	0	2	5
Diuisor	b . . .	6							
Rad. sing. prima . . $b = 6$									
Ablatitium	bb . . .	3	6						
Radix singularis			6						
Homogeneum reliquum resoluendum		1	2	2	3	3	0	2	5

Radix singularis			6
Homogenei reliquum resoluendum			4 2 2 3 3 0 2 5
Diuisor		2.*b*	1 2 0
Radix sing. decuplata	*b* = 60	2.*bc*	1 0 8 0
Rad. sing. secunda	*c* = 9	*cc*	8 1
Ablatitium		*Bc*	1 1 6 1
Radix aucta			6 9
Homogenei reliquum resoluendum			6 2 3 0 2 5
Diuisor		2.*b*	1 3 8 0
Rad. aucta decupl.	*b* = 690	2.*bc*	5 5 2 0
Rad. sing. tertia	*c* = 4	*cc*	8 1
Ablatitium		*Bc*	5 5 3 6
Radix aucta			6 9 4
Homogenei reliquum resoluendum			6 9 4 2 5
Diuisor		2.*b*	1 3 8 8 0
Rad. aucta decupl.	*b* = 6940	2.*bc*	6 9 4 0 0
Rad. sing. quarta	*c* = 5	*cc*	2 5
Ablatitium		*Bc*	6 9 4 2 5
Rad. vniuersalis completè educta			6 9 4 5
Homogenei reliquum finale			0 0 0 0 0

E dato igitur homogeneo 48233025. factâ ipsius ad hunc modum resolutione educta est radix 6945. radici quæsititiæ *a*. æqualis quæ educenda erat.

Lemma.

Quoniam proposita æquatio *aa* = 48233025. de radice 6945. per resolutionem educta retrogradâ compositionis viâ explicabilis est: E conuerso igitur per æquationis explicationem compositiuè factam, quæ in præsenti exemplo satis obuia est, tam ipsius resolutionis quàm Canonis cuius directione facta est resolutio, veritas vt oportuit comprobatur.

Nota 1.

In Problematis hisce quæ ad Exegesin numerosam spectant, in numeris proposita intelligenda est æquatio, cuius homogeneum datum, si simplex sit vel si affecta vna cum homogeneo coefficientia data numeris exprimuntur.

Nota 2.

Prætérea hoc quoque imprimis notadnum est, duarum notarum *Ab. Bc.* quæ in Canone

Canone constituendo ad speciei canonicæ particularia bifariam distinguenda adscribuntur, primam, scilicet *Ab.* particularia ea quæ ad primæ radicis singularis eductionem pertinent, denotare intelligendum est, secundam verò *Bc.* ad particularia quæ secundariarum radicum singularium eductioni inseruiunt denotanda, perpetuâ iteratione adhibendam esse.

Quod sic porro accipiendum est, duas istas notas *Ab. Bc.* quadripartitò rursus distribuendas esse, videlicet *A.* pro diuisore primario significando, ipsam verò *Ab.* pro ablatitio, atque *B.* pro diuisoribus secundarijs, *Bc.* pro ablatitijs. Et quatuor hisce notas inter ipsius canonis particularia seriatim suis locis inserendas.

Notarum autem istarum significationem & vsum pro diuersa particularium quibus significandis adijciuntur affectione vel numero diuersificari necesse est. Nam etsi in æquationibus simplicibus resoluendis in quibus diuisores & ablatitia primaria ex vnico particulari constant vlterior præter ipsius canonis notas, notarum *A.* & *Ab.* ad diuisores & ablatitia designanda adscriptio superuacanea foret, generaliter tamen vbi diuisores & ablatitia ex pluribus particularibus componuntur, si sint eiusdem affectionis notarum *A. Ab. B. Bc.* ad particularium summas, si vero contrariarum affectionũ ad eorum differentias, absque molesta totius fere canonis rescriptione (quod alias fieret) denotandas admodum commodus est vsus. Hæc licet in sequentium exemplorum schematismis vel minimum aduertenti manifesta sint, non tamen abs re visum est hoc loco adnotasse.

PROBLEMA 2.

E dato æquationis . . . $aa + da = ff$. . . . in numeris propositæ homogeneo radicem radicis quæsititiæ *a.* valorem analyticè educere.

Sit æquatio numerosè proposita $aa + 432.\,a = 13584208.$

Ergo . . . . . . $432 = d.$ & $13584208 = ff.$

Ponatur . . . . $b + c = a.$

Ergo . . . . . . $\left.\begin{array}{c} b+c \\ b+c \end{array}\right| + \left.\begin{array}{c} d \\ b+c \end{array}\right| = 13584208$

Factis igitur & bifariam adhunc modum distributis homogeneis particularibus, fit . . . . . .

$$\underbrace{\begin{array}{l} +db \\ +bb \end{array}}_{Ab} \quad \underbrace{\begin{array}{l} +dc \\ +2.bc \\ +.cc \end{array}}_{Bc} = 13584208.$$

Est autem æquationis huius pars speciosa bipartita *Ab. Bc.* resolutionis canon cuius applicatione operis analytici processus dirigendus est, quem rite constitutum esse ex sequenti Lemmate constabit.

Factâ igitur canonis huius applicatione omnino vt in subiecto exempli schematismo ordinata conspicitur, fiat ipsius directione homogenei dati 13584208. ad radicem ex eo educendam resolutio, vt sequitur.

Radix

Radix vniuersalis succeßiue educenda 3 4 7 6

Homogeneum resoluendum *ff* 1 3 5 8 4 2 0 8

d 0 4 3 2

Radix singularis *b* == 3 *b* 3

Diuisor *A* 3 4 3 2

db 1 2 9 6

Rad. sing. prima *b* == 3 *bb* 9

Ablatitium *Ab* 1 0 2 9 6

Radix singulalis 3

Homogenei reliquum resoluendum 3 2 8 8 2 0 8

d 4 3 2

Rad. sing. decuplata *b* == 30 2.*b* 6 0

Diuisor *B* 6 4 3 2

dc 1 7 2 8

Rad. sing. secenda *c* == 4 2.*bc* 2 4 0

cc 1 6

Ablatitium *Bc* 2 7 3 2 8

Radix aucta 3 4

Homogenei reliquum resoluendum 5 5 5 4 0 8

d 4 3 2

Rad. aucta decupl. *b* == 340 2.*b* 6 8 0

Diuisor *B* 7 3 3 2

dc 3 0 2 4

Rad sing. tertia *c* == 7 2.*bc* 4 7 6 0

cc 4 9

Ablatitium *Bc* 5 1 1 1 4

Radix aucta. 3 4 7

Homogenei reliquum resoluendum 4 4 2 6 8

d 4 3 2

Rad. aucta decupl. *b* == 3470 2.*b* 6 9 4 0

Diuisor *B* 7 3 7 2

dc 2 5 9 2

Rad. sing. quarta *c* == 6 2.*bc* 4 1 6 4 0

cc 3 6

Ablatitium *Bc* 4 4 2 6 8

Radix vniuersalis complete educta 3 4 7 6

Homogenei reliquum finale 0 0 0 0 0

E dato

E dato igitur homogeneo 13584208. factâ ipsius ad hunc modum resolutione educta est radix 3476. radici quæsititiæ a. æqualis, quæ ex intento Problematis educenda erat.

Lemma

Si è dato propositæ æquationis $aa + 432.\ a = 13584208.$ homogeneo radix 3476. resolutionis viâ educta, radici quæsititiæ a. æqualis, & æquationis explicatoria sit,

est . . . . $aa = \dfrac{3476.}{3476}$ & $432.\ a = \dfrac{432}{3476}$

Sed . . . . $\dfrac{3476}{3476} = 12082576.$ & . . $\dfrac{432}{3476} = 1501632.$

Et . . . . . $\begin{matrix} + 12092576 \\ + 1501632 \end{matrix} = 13584208$

Ergo . . . . . . $aa + 432.\ a = 13584208$

Est autem ipsa æquatio proposita.

Congrua est igitur æquationis de radice 3476. retrogradâ compositionis viâ facta explicatio, ac proinde radix eductitia 3476. radici quæsititiæ a. æqualis est, & resolutio per quam educta est radix vere facta est, & consequenter, canon cuius directione facta est resolutio, rite constitutus. Quod inprimis probasse oportuit.

Casus deuolutionis.

In æquationis $aa + da = ff.$ ad numeros reuocatæ resolutione interdum accidit coefficiens in anteriora eo vsque extendi, vt ab homogeneo resoluendo auferri non possit. In tali casu ad proximè succedens punctum vel tertium, vel vlterius si opus fuerit, deuoluendum est coefficiens, donec diuisioni & operis inceptioni locus sit, quod sequentibus duobus exemplis declaratur.

Deuolutionis exemplum I.

Æquatio resoluenda . . . . $\left\{ \begin{matrix} aa + da = ff. \\ aa + 75325.\ a = 41501984 \end{matrix} \right.$

Resolutionis canon . . . . $\left\{ \begin{matrix} + db & + ..dc \\ + bb & + 2.bc \\ \overline{Ab} & + ..cc \\ & \overline{Bc} \end{matrix} \right.$

Radix vniuersalis successiuè educenda					5		4		7
Homogeneum resoluendum	ff . . .	4	1	5	0	1	9	8	4

Radix vniuersalis successiuè educenda			5 4 7
Homogeneum resoluendum.		ff	4 1 5 0 1 9 8 4
		d	7 5 3 2 5
		b	3
Diuisor		A	7 5 3 2 5
		db	3 7 6 6 2 5
Rad. sing. prima	$b = 5$	bb	2 5
Ablatitium		Ab	3 7 9 1 2 5
Radix singularis			5
Homogenei reliquum resoluendum			3 5 8 9 4 8 4
		d	7 5 3 2 5
Rad.sing.decuplata	$b = 50$	$2.b$	1 0 0
Diuisor		B	7 6 3 2 8
		dc	3 0 1 3 0 0
Rad.sing.secunda	$c = 4$	$2.bc$	4 0 0
		cc	1 6
Ablatitium		Bc	3 0 5 4 6 0
Radix aucta			5 4
Homogenei reliquum resoluendum			5 3 4 8 8 4
		d	7 5 3 2 3
Rad. aucta decupl.	$b = 540$	$2.b$	1 0 8 0
Diuisor		B	7 6 4 0 5
		dc	5 9 7 2 7 7
Rad sing. tertia	$c = 7$	$2.bc$	4 7 6 0
		cc	4 9
Ablatitium		Bc	5 3 4 8 8 4
Radix vniuersalis completè educta			5 4 7
Homogenei reliquum finale			0 0 0 0 0

E dato igitur homogeneo 41501984. factâ ipsius ad hunc modum resolutione educta est radix 547. radici quæsititiæ a. æqualis, quæ ex intento Problematis educenda erat.

Deuolutionis exemplum 2.

Æquatio resoluenda . . . . $\begin{cases} aa + da = ff. \\ aa + 675325.\ a = 369701984. \end{cases}$

Canon

Canon resolutionis . . . . . $\left\{ \begin{array}{ll} +\,db & +\,..\,dc \\ +\,bb & +\,2.\,bc \\ \hline Ab & +\,..\,cc \\ & \overline{Bc} \end{array} \right.$

Radix vniuersalis successiuè educenda							5		4		7
Homogeneum resoluendum		*ff*	3	6	9	7	0	1	9	8	4
		d		6	7	5	3	2	5		
		b					5				
Diuisor		*A*		6	7	5	8	2	5		
		db	3	3	7	6	6	2	5		
Radix singularis prima	*b* = 5	*bb*				2	5				
Ablatitium		*Ab*	3	3	7	9	1	2	5		
Radix singularis							5				
Homogenei reliquum resoluendum				3	1	7	8	9	4	8	4
		d			6	7	5	3	2	5	
Rad. sing. decuplata	*b* = 50	2.*b*					1	0	0		
Diuisor		*B*			6	7	6	3	2	5	
		dc		2	7	0	1	4	0	0	
Radix sing. secunda	*c* = 4	2.*bc*					4	0	0		
		cc						1	6		
Ablatitium		*Bc*		2	7	0	5	4	6	0	
Radix aucta							5		4		
Homogeneum reliquum resoluendum					4	7	3	4	8	8	4
		d				6	7	5	3	2	5
Rad. aucta decupl.	*b* = 540	2.*b*						1	0	8	0
Diuisor		*B*				6	7	6	4	0	5
		dc			4	7	2	7	2	7	5
Rad. sing. quarta	*c* = 6	2.*bc*						7	5	6	0
		cc								4	9
Ablatitium		*Bc*			4	7	3	4	8	8	4
Rad. vniuersalis completè educta							5		4		7
Homogenei reliquum finale					0	0	0	0	0	0	0

E dato igitur homogeneo 369701984. factâ ipsius ad hunc modum resolutione educta est radix 547. radici quæsititiæ *a.* æqualis, quæ ex intento Problematis educenda erat.

PRO-

PROBLEMA 3.

E dato æquationis . . . $aa - da$ ═══ ff. . . in numeris propositæ homogeneo radicem radicis quæsititiæ a. valorem analyticè educere.

Sit æquatio numerosè proposita $aa - 624.\ a$ ═══ 16305156.

Ergo . . . . . 624 ═══ d. & 16305156 ═══ ff.

Ponatur . . . . $b + c$ ═══ a.

Ergo . . . . . . $\left.\begin{matrix} b + c \\ b + c \end{matrix}\right|$ $\left.\begin{matrix} - d \\ b + c \end{matrix}\right|$ ═══ 16305156

Factis igitur & bifariam ad hunc modum dispositis homogeneis particularibus, fit . . . . . $\begin{matrix} - db \\ + bb \\ \hline Ab \end{matrix}$ $\begin{matrix} - dc \\ + 2.bc \\ + .cc \\ \hline Bc \end{matrix}$ ═══ 16305156.

Est autem æquationis huius pars speciosa bipartita Ab. Bc. resolutionis canon cuius applicatione operis analytici processus dirigendus est, quem rite constitutum esse per explicationis congruentiam superioris Lemmatis exemplo manifestari potest.

Factâ igitur canonis huius applicatione omnino vt in subiecto exempli schematismo ordinata conspicitur, fiat ipsius directione homogenei dati 16305156. ad radicem ex eo educendam resolutio, vt sequitur.

Rad. vniuersalis sucessiuè educenda		4 3 6 2
Homogeneum resoluendum	ff	1 6 3 0 5 1 5 6
	$-d$	— 6 2 4
	b	4
Diuisor	A	3 3 7 6
Radix sing. prima b ═══ 4	$-db$	— 2 4 9 6
	bb	1 6
Ablatitium	Ab	1 3 5 0 4
Radix singularis		4
Homogenei reliquum resoluendum		2 8 0 1 1 5 6
	$-d$	— 6 2 4
Rad. sing decupl. b ═══ 40	$2.b$	8 0
Diuisor	B	7 3 7 9
	$-dc$	— 1 8 7 2
Rad. sing. secunda c ═══ 3	$2.bc$	2 4 0
	cc	9
Ablatitium	Bc	2 3 0 2 8
Radix aucta		4 3
Homogenei reliquum resoluendum		4 9 8 4 5 6

Radix

Radix aucta.		4 3
Homogenei reliquum resoluendum		4 9 8 3 5 6
	$-d$	6 2 4
Rad. aucta decupl. $b = 430$	$2.b$	8 6 0
Diuisor	B	7 9 7 6
	$-dc$	3 7 4 4
Rad. sing. tertia. $c = 6$	bc	5 1 6 0
	cc	3 6
Ablatitium	Bc	4 8 2 1 6
Radix singularis		4 3 6
Homogenei reliquum resoluendum		1 6 1 9 6
	$-d$	6 2 4
Rad. aucta decupl. $b = 4360$	$2.b$	8 7 2 0
Diuisor	B	8 0 9 6
	$-dc$	1 2 4 8
Rad. sing. quarta $c = 2$	$2.bc$	1 7 4 4 0
	cc	4
Ablatitium	Bc	1 6 1 9 6
Rad vniuersalis completè educta		4 3 6 2
Homogenei reliquum finale		0 0 0 0 0

E dato igitur homogeneo 16305156. factâ ipsius ad hunc modum resolutione educta est radix 4362. radici quæsititiæ *a.* æqualis, quæ ex intento Problematis educenda erat.

Casus anticipationis.

In æquatione $aa - da = ff.$ numerosè propositâ resoluendâ, accidit interdum vt coefficiens diuisorium pluribus abundet singulis figuris quàm homogeneum resoluendum binis. Itaque vt resolutioni sit locus, præponatur homogeneo ad læuam ea ciphrarum multitudo vt illud tot puncta quadratica recipiat quot habet coefficiens simplices figuras, & ad primum punctum vacuum tanquam per anticipationẽ, opus resolutionis inchoetur, in quo hoc inest compendij, vt prima coefficientis figura primæ radici singulari educendæ aut æqualis sit, aut ea proximè minor.

Anticipationis exemplum.

Æquatio resoluenda .... $\begin{cases} aa - da = ff. \\ aa - 6253.\ a = 6254. \end{cases}$

Canon resolutionis ..... $\begin{cases} -db & -..dc \\ +bb & +2.bc \\ \overline{Ab} & +..cc \\ & \overline{Bc} \end{cases}$

Radix vniuerſalis ſucceſſiuè educenda		6 2 5 4
Homogeneum reſoluendum	*ff*	0 0 0 6 2 5 4
	— *d*	— 6 2 5 3
	b	6
Diuiſor	*A*	2 5 3
Radix ſingularis prima . *b* ═ 6	— *db*	— 3 7 5 1 8
	bb	3 6
Ablatitium	*Ab*	1 5 1 8
Radix ſingularis		6
Homogenei reliquum reſoluendum		1 5 2 4 2 5 4
	— *d*	— 6 2 5 3
Rad. ſing decuplata . . *b* ═ 50	2. *b*	1 2 0
Diuiſor	*B*	5 7 4 7
Radix ſing. ſecunda . . *c* ═ 2	— *dc*	— 5 7 5 0 6
	2. *bc*	2 4 0
	cc	4
Ablatitium	*Bc*	1 1 8 9 4
Radix aucta		5 4
Homogeneum reliquum reſoluendum		3 3 4 8 6 4
	— *d*	— 6 2 5 3
Rad. aucta decupl. *b* ═ 620	2. *b*	1 2 4 0
Diuiſor	*B*	6 1 4 7
Rad. ſing. tertia . . *c* ═ 5	— *dc*	— 3 1 2 6 5
	2. *bc*	6 2 0 0
	cc	2 5
Ablatitium	*Bc*	3 0 9 8 5
Radix aucta		6 2 5
Homogenei reliquum reſoluendum		2 5 0 0 4
	— *d*	— 6 2 5 3
Rad. ſing. decuplata *b* ═ 6250	2. *b*	1 2 5 0 0
Diuiſor	*B*	6 2 4 7
Rad. ſing. quarta *c* ═ 4	— *dc*	— 2 5 0 1 2
	2. *bc*	5 0 0 0 0
	cc	1 6
Ablatitium	*Bc*	2 5 0 0 4
Radix vniuerſalis completè educta		6 2 5 4
Homogenei reliquum finale		0 0 0 0 0

E dato

E dato igitur homogeneo 6254. factâ ipsius ad hunc modum resolutione educta est radix 6254. radici quæsititiæ a. æqualis, quæ ex intento Problematis educenda erat.

Casus rectificationis.

Prætereà in æquatione $aa - da = ff$. numerosè propositâ accidit non rarò coefficiens eousque in anteriora pertingere vt primæ radicis singularis electio per illius confusionem reddatur dubia. Quare magis expedit vt eo casu coefficientis quadratum homogeneo resoluendo addatur & summæ illius radix singularis prima pro primâ singulari homogenei resoluendi sumatur, quæ vel consentanea erit, vel consentaneæ proximè minor. Huiusmodi eductionis artificium Epanorthôsin (Authore *Vieta*) seu rectificationem appellare licet.

In æquationibus etiam affirmatis si similis dubitationis casus in primæ radicis electione occurrat, simile rectificationis remedium adhiberi potest. In his autem non summæ sed differentiæ quadratici coefficientis & homogenei resoluendi radix singularis primâ pro prima singulari homogenei resoluendi sumenda est; quæ etiam vel consentanea erit, vel consentaneæ proximè minor.

Æquatio resoluenda . . . . $\begin{cases} aa - da = ff. \\ aa - 732.\,a = 86005. \end{cases}$

Resolutionis canon . . . . $\begin{cases} -db \\ +bb \\ \hline Ab \end{cases}$ $\begin{matrix} -..dc \\ +2.bc \\ +..cc \\ \hline Bc \end{matrix}$

Exemplum rectificationis.

Homogeneum datum	ff		8	6	0	0	5
Coefficietnis quadrat.	dd	5	3	5	8	2	4
Summa	$ff+dd$	6	2	1	8	2	9
Rad.sing.prima		8					

Resolutio continuata.

Radix vniuersalis successiuè educenda			8		3		5
Homogen. resoluendum	ff		8	6	0	0	5
$b = 8$	$-d$		− 7	3	2		
	$-db$	− 5	8	5	6		
	bb	6	4				
Ablatitium.	Ab		5	4	4		
Radix singularis prima			8				
Homogenei residuum resoluendum			3	1	6	0	5

Radix

Radix singularis prima			8
Homogenei residuum resoluendum			3 1 6 0 5
	$b = 80$	$-d$	7 3 2
		$2.b$	1 6 0
Diuisor		B	8 6 8
		$-dc$	2 1 9 9
	$c = 3$	$2.bc$	4 8 0
		cc	9
Ablatitium		Bc	2 6 9 4

Radix aucta			8 3
Homogenei residuum resoluendum			3 6 6 5
		$-d$	7 3 2
	$b = 830.$	$2.b$	1 6 6 0
Diuisor		B	9 2 8
		$-dc$	3 6 6 0
	$b = 5$	$2.bc$	8 3 0 0
		cc	3 5
Ablatitium		Bc	4 6 6 5

Radix completè educta	8 3 5
Homogenei residuum nullum	0 0 0 0

PROBLEMA 4.

E dato æquationis $aa + da = ff$. quæ de duplici radice explicabilis est, in numeris propositæ homogeneo radicem radicis quæsititiæ a. æqualem vtramque analyticè educere.

Æquatio resoluenda $\begin{cases} -aa + da = ff. \\ -aa + 370.\ a = 9261. \end{cases}$

Resolutionis canon $\begin{cases} + db + dc \\ --bb - 2.bc \\ \overline{Ab} \quad - .cc \\ \qquad \overline{Bc} \end{cases}$

Eductio radicis minoris.

Rad.	2 7
Homogeneum resoluendum	9 2 6 1

Rad.

Rad.		2 7
Homogeneum resoluendum		9 2 6 1
b = 2	d	3 7 0
	$-b$	2
Diuisor	A	3 5 0
b = 2	db	7 4 0
	$-bb$	4
Ablatitium	Ab	7 0 0

Rad.		2
Homogenei residuum resoluendum		2 2 6 1
b = 20	d	3 7 1
	$-2.b$	4 0
Diuisor	B	3 3 0
	dc	2 5 9 0
c — 7	$2.bc$	2 8 0
	$-cc$	4 9
Ablatitium	Bc	2 2 6 1

Rad.	2 7
Homogen. residuum nullum	0 0 0 0

Eductio radicis maioris per anticipationem.

Rad.		3 4 3
Homogeneum resoluendum		0 9 2 6 1
	d	3 7 0
b = 3	$-b$	3
Diuisor	A	7 0
	db	1 1 1 0
b = 3	$-bb$	9
Ablatitium	Ab	2 1 0

Rad.	3
Homogenei residuum resoluendum	1 1 7 3 9

Rad.			3
Homogenei reliquum resoluendum			— 1 1 7 3 9
		d	3 7 0
	b = 30	— 2.*b*	— . 6 0
Diuisor		*B*	— 2 3 0
		dc	1 4 8 0
	c = 4	— 2.*bc*	— 2 4 0
		— *cc*	— . 1 6
Ablatitium		— *Bc*	— 1 0 8 0
Rad.			3 4
Homogenei reliquum resoluendum			. 9 3 9
		d	3 7 0
	b = 340	— 2.*b*	— . . . 6 8 0
Diuisor		*B*	— . . 3 1 0
		dc	1 1 1 0
	c = 3	— 2.*bc*	— . 2 0 4 0
		— *cc*	— 9
Ablatitium		— *Bc*	— . 9 3 9
Rad.			3 4 3
Homogenei reliquum finale			0 0 0 0

E dato igitur homogeneo *9261*. factâ ipsius ad hunc modum duplici resolutione eductæ sunt radices duæ 27. & 243. radici quæsititiæ *a*. æquales, quæ educendæ erant.

Compendium.

Per Theorem. 2. Sect. 5. summa duarum radicum de quibus proposita æquatio — *aa* + *da* = *ff*. explicabilis est, dato coefficieuti, & factum ex ipsis dato homogeneo æquatur, vt in exemplo numeroso 27 + 343 = 370. & 27 | 343 = *6261*. vnde inuenta vna altera absque opere analytico exhibetur, quod pro compendio esse potest.

Nota 1.

Æquationum quadraticarum resolutionem in veteri praxi apodicticè tractari posse notum est. *Vieta* tamen ne propriæ inuentionis existimationi quadraticarum mutilatione derogaret, exegeticen suam numerosam artem naturâ generalem generali ac integrâ methodo concinnatam in publicum prodire voluit. Cuius exemplo Analysta noster quadraticarum quoque Exegesin numerosam in scriptis suis proposuit. Hinc est, quod de Deuolutionis & Anticipationis & Rectificationis

ficationis regulis quæ ad artem generalem spectant in quadraticis hisce primò ex methodi necessitate præcipiendum erat.

Nota 2.

Notandum hic quoque venit, quod per radicum quæ ad diuisores secundarios constituendos adhibentur decuplationem (vt in antecedentibus schematismis fit, & in sequentibus obseruandum est) tam diuisoris quam ablatitij particularia ad puncta directoria æquationis gradualibus congruè designata vniformiter terminantur. Vnde particularium ordinatio prompta & facilis euadit, quæ in forma antehac præscripta & visitata, particularium terminationibus variatis, nimium curiosa facta est & anxia.

Nota 3.

Præterea aduertendum est, titulos in exemplorum schematismis ad marginem adscriptos, non ad operis Analytici necessitatem sed ad canonis applicati, & radicum notas designandas solummodo adhibitos esse. Quod in nouitiâ hac artis instructione opportunè faciendum erat. In praxi autem reali, quæ simplici canonis, continuatâ serie, applicatione sufficientissimè dirigitur, verboso huiusmodi apparatu non erit opus. Quod hoc loco obiter admonuisse sufficiat.

Ad Æquationes cubicas resoluendas.

PROBLEMA 5.

E dato æquationis cubicæ simplicis . . . $aaa = ggg$ . . . in numeris propositæ hmogeneo radicem radicis quæsititiæ a. valorem analyticè educere.

Sit æquatio numerosè proposita . . . $aaa = 105689636352$.

Et ponatur $b + c = a$.

Erit inde . . . .

$$\left.\begin{array}{l} b+c \\ b+c \\ \underline{b+c} \end{array}\right| = 105689636352.$$

Factis igitur & ordinatis particularibus.

fit . . . .

$$\underbrace{+\,bbb}_{Ab} \quad \begin{array}{l} +\,3.\,bbc \\ +\,3.\,bcc \\ \underbrace{+\,.\,ccc}_{Bc} \end{array} = 10568963636352.$$

Est autem species ista bipartita operationis analyticæ canonica seu directoria, prima pars Ab. pro prima radice, secunda verò Bc. pro secundarijs, vt in subiecto schematismo patet.

Fiat igitur ipsius directione è dato homogeneo 105689636352. radicis eductio, vt sequitur.

Rad.

Radix vniuersalis successiuè educenda				4			7			2			8
Homogeneum resoluendum		1	0	5	6	8	9	6	3	6	3	5	2
Diuisor	bb		1	6									
Rad. sing prima $b = 4$													
Ablatitium	bbb		6	4									
Radix singularis prima				4									
Homogenei residuum resoluendum			4	1	6	8	9	6	3	6	3	5	2
Diuisor	$3.bb$			4	8	0	0						
Radix sing. dupl. $b = 40$	$3.bb$		3	3	6	0	0						
Rad. sing. secund. $c = 7$	bcc			5	8	8	0						
	ccc				3	4	3						
Ablatitium	Bc		3	9	8	2	3						
Radix aucta				4			7						
Homogenei residuum resoluendum				1	8	6	6	6	3	6	3	5	2
Diuisor	$3.bb$				6	6	2	7	0	0			
	$3.bbc$			1	3	2	5	4	0	0			
Rad. aucta decuplata $b = 470$	$3.bcc$						5	6	4	0			
Radix singula. tertia $c = 2$	ccc									8			
Ablatitium	Bc			1	3	3	1	0	4	8			
Radix aucta				4			7			2			
Homogeneum reliquum resoluendum					5	3	5	5	8	8	3	5	2
Diuisor	$3.bb$					6	6	8	3	5	2	0	0
	$3.bbc$				5	3	4	6	8	1	6	3	0
Rad. aucta decupl. $b = 4720$	$3.bcc$							9	9	6	2	4	0
Rad. sing. quarta $c = 8$	ccc										5	1	2
Ablatitium	Bc				5	3	5	5	8	8	3	5	2
Radix vniuersalis completè educta				4			7			2			8
Homogenei reliquum finale					0	0	0	0	0	0	0	0	0

E dato igitur homogeneo 105689636352. factâ ipsius ad hunc modum resolutione educta est radix 4728. radici quæsititiæ a. æqualis, quæ ex intento problematis educenda erat.

PROBLEMA 6.

E dato æquationis .... $aaa + daa + ffa = ggg$ ... in numeris propositæ homogeneo radicem radicis quæsititiæ a. valorem analyticè educere.

Sit

Sit Æquatio numerosè proposita . aaa . $+ 68.aa + 4352.a$ ═══ 186394079.

Ergo . . 68 ═══ d . ; & 4352 ═══ ff. . . & 186394079 ═══ ggg.

Ponatur . . . . $b + c$ ═══ a.

Ergo . . . . . $$\begin{array}{l|l|l|} b+c & +d & +ff \\ b+c & b+c & b+c \\ b+c & b+c & \end{array}$$ ═══ 186394079.

Factis igitur homogeneis particularibus,

fit . . . . $$\begin{array}{llll} +..bbb & +\ .\ dbb & +ffb & \\ +3.bbc & +2.dbc & +ffc & \\ +3.bcc & +\ .\ dcc & & \\ +..ccc & & & \end{array}$$ ═══ 186394079.

Et ijsdem bifariam ad hunc modum distributis,

fit . . . . . $$\underbrace{\begin{array}{l} +ffb \\ +dbb \\ +bbb \end{array}}_{Ab} \quad \underbrace{\begin{array}{ll} +ffc & +3.bbc \\ +dcc & +3.bcc \\ +2.dbc & +..ccc \end{array}}_{Bc}$$ ═══ 186334079.

Est autem æquationis huius pars speciosa bipartita Ab. Bc. resolutionis Canon cuius applicatione operis analytici processus dirigendus est. Quem rite constitutum esse ex sequenti Lemmate constabit.

Facta igitur Canonis huius applicatione omnino vt in subiecti exempli schematismo ordinata cospicitur, fiat ipsius directione dati homogenei 186394079. ad radicem ex eo educendam resolutio, vt sequitur.

Radix vniuersalis successiuè educenda				5			4			7
Homogen. resoluendum	ggg	1	8	6	3	9	4	0	7	9
	ff . . .			0	4	3	5	2		
	d . . .			0	6	8				
	bb . .		2	5						
Diuisor	A . .		2	6	1	1	5	2		
	ffb . . .			2	1	7	6	0		
Rad. singularis prima b ═══ 5	dbb . . .		1	7	0	0				
	bbb .	1	2	5						
Ablatitium	Ab . .	1	4	4	1	7	6	0		
Radix singularis				5						
Homogenei reliquum resoluendum			4	2	2	1	8	0	7	9

Radix singularis		5
Homogenei residuum resoluendum		4 2 2 1 8 0 7 9

	ff	4 3 5 2
	d	6 8
Radix singularis decuplata $b = 50$	2.*db*	6 8 0 0
	3.*bb*	7 5 0 0
	3.*b*	1 5 0
Diuisor	*B*	8 3 8 0 3 2
	ffc	1 7 4 0 8
	dcc	1 0 8 8
	2.*dbc*	2 7 2 0 0
Rad. singularis secunda $c = 4$	3.*bbc*	3 0 0 0 0
	3.*bcc*	2 4 0 0
	ccc	6 4
Ablatitium	*Bc*	3 5 4 6 6 8 8

Radix aucta		5 4
Homogenei reliquum resoluendum		6 7 5 1 1 9 9

	ff	4 3 5 2
	d	6 8
Radix aucta decuplata $b = 540$	2.*db*	7 3 4 4 0
	3.*bb*	8 7 4 8 0 0
	3.*b*	1 6 2 0
Diuisor	*B*	9 5 4 2 8 0
	ffc	3 0 4 6 4
	dcc	3 3 3 2
	2.*dbc*	5 1 4 0 8 0
Radix singularis tertia $c = 7$	3.*bbc*	6 1 2 3 6 0 0
	3.*bcc*	7 9 3 8 0
	ccc	3 4 3
Ablatitium	*Bc*	6 7 5 1 1 9 9

Radix vniuersalis completè educta		5 4 7
Homogenei reliquum finale		0 0 0 0 0 0 0 0

E dato igitur homogeneo 186394079. factâ ipsius ad hunc modum resolutione educta est radix 547. radici quæsititiæ *a.* æqualis, quæ ex intento Problematis educenda erat.

Lemma.

Quoniam proposita æquatio . . . *aa* + 68. *aa* + 4352. *a* = 186394079. de radice 547. per resolutionem educta, retrogradâ compositionis viâ explicabilis est. E conuerso

uerso igitur per æquationis explicationem compositiue factam, tam ipsius resolutionis quam Canonis cuius directione facta est resolutio veritas vt oportuit asseritur. Generalis est verificationis huiusmodi ratio, & ad sequentia Problemata vel actu adscribenda vel vt necessaria subintelligenda.

Problematis & exempli 6. Schematismus alius variatâ nonnihil canonis ordinatione.

Radix vniuersalis successiuè educenda		5 4 7
Homogeneum resoluendum	ggg	1 8 6 3 9 4 0 7 9
	ff	4 3 5 2
	db	3 4 0
	bb	2 5
Diuisor	A	2 8 8 3 5 2
	ffb	2 7 6 0 0
Rad. sing. prima $b = 5$	dbb	1 7 0 0
	bbb	1 2 5 5
Ablatitium	Ab	1 4 4 6 7 6 0
Radix singularis		5
Homogenei residuum resoluendum		4 2 2 1 8 0 7 9
	ff	4 3 5 2
Radix sing. dupl. $b = 50$	2.db	6 8 0 0
	3.bb	7 5 0 0
Diuisor	B	8 2 2 3 5 2
	ffc	1 7 4 0 8
	2.dbc	2 7 2 0 0
	3.bbc	3 0 0 0 0
Rad. sing. secund. $c = 4$	dcc	1 0 8 8
	3.bcc	2 4 0 0
	ccc	6 4
Ablatitium	Bc	3 5 4 6 6 8 8
Radix aucta		5 4
Homogenei residuum resoluendum		6 7 5 1 1 9 9

Radix

Radix aucta		5			4			
Homogenei reliquum resoluendum		6	7	5	1	1	9	9
	ff				4	3	5	2
Rad. aucta decuplata $b = 540$	2.*db*			7	3	4	4	0
	3.*bb*		8	7	4	8	0	0
Diuisor	*B*		9	5	2	5	9	2
	ffc			3	6	4	6	4
	2.*dbc*		5	1	4	0	8	0
	3.*bbc*	6	1	2	3	6	0	0
Radix singula. tertia $c = 7$	*dcc*				3	3	3	2
	3.*bcc*			7	9	3	8	0
	ccc					3	4	3
Ablatitium	*Bc.*	6	7	5	1	1	9	9
Radix singularis completè educta		5			4			7
Homogenei reliquum finale		0	0	0	0	0	0	0

Nota

In duobus hisce exemplis quæ ad eiusdem æquationis resolutionem pertinent diuersa canonis ordinatio diuisoribus applicata cernitur. In priore propter nonnullam in diuisore constituendo veritati approximationem particularia heterogenea ad diuisorem componendum promiscue assumuntur. Quam diuisionis formam diuisore ex partibus gradualibus seu scansorijs aggregato *Vieta* climacticam appellat. In secundo vero ad homogeniæ legem seruandam paulò difficiliore tentamine proceditur. In sequentibus igitur climactica forma vt magis expedita vsurpatur. Nam præter iam dictum discrimen in climactica diuisione diuisoris componentia ablatitij componentibus, numero & ordine ijs respondentia, quodammodo præparatoria sunt. Quod compendij alicuius instar, in praxi esse reperitur.

PROBLEMA 7.

E dato æquationis . . . . $aaa + ffa = ggg$ . . . . in numeris propositæ homogeneo radicem radici quæsititiæ *a.* æqualem analicè educere.

Sit æquatio numerosè proposita . . . . $aaa + 45796.\, a = 449324752.$

Ergo . . . . $45796 = ff.$ & $449324752 = ggg.$

Ponatur . . . . . $b + c = a.$

Ergo . . . . . . $\begin{array}{l|l} b+c & +ff \\ b+c & +b+c \\ b+c & \end{array} = 449324752.$

Factis

Factis igitur homogeneis particularibus,

$$\text{fit} \quad \begin{array}{l} + \ldots bbb \\ + 3.bbc \\ + 3.bcc \\ + \ldots ccc \end{array} \quad \begin{array}{l} + ffb \\ + ffc \\ \\ \\ \end{array} = 449324752.$$

Et ijsdem bifariam ad hunc modum distributis,

$$\text{fit} \quad \underbrace{\begin{array}{l} + ffb \\ + bbb \end{array}}_{Ab} \quad \underbrace{\begin{array}{l} + \ldots ffc + 3.bcc \\ + 3.bbc + \ldots ccc \end{array}}_{Bc} = 449324752.$$

Est autem æquationis huius pars speciosa bipartita *Ab. Bc.* resolutionis canon cuius applicatione operis analytici processus dirigendus est, quem ritè constitutum esse ex sequenti Lemmate constabit.

Facta igitur canonis huius applicatione omnino vt in subiecti exempli schematismo ordinata conspicitur, fiat ipsius directione dati homogenei 449324752. ad radicem ex eo educendam resolutio, vt sequitur.

Rad.		7 . . 4 . . 6
Homogeneum resoluendum	ggg	4 4 9 3 2 4 7 5 2
	ff	4 5 7 9 6
	bb	4 9
Diuisor	*A*	5 3 5 7 9 6
	ffb	3 2 0 5 7 2
Radix singularis prima $b = 7$	*bbb*	3 4 3
Ablatitium	*Ab*	3 7 5 0 5 7 2
Radix singularis		7
Homogenei residuum resoluendum		7 4 2 6 7 5 5 2
	ff	4 5 7 9 6
Radix singularis decupl. $b = 70$	3.*bb*	1 4 7 0 0
	3.*b*	2 1 0
Diuisor	*B*	1 5 3 6 7 9 6
	ffc	1 8 3 1 8 4
	3.*bbc*	5 8 8 0 0
Rad. singul. secunda $c = 4$	3.*bcc*	3 3 6 0
	ccc	6 4
Ablatitium	*Bc*	6 4 0 5 5 8 4
Radix aucta		7 . . 4
Homogenei residuum resoluendum		1 0 2 1 1 7 1 2

Radix aucta			7			4			
Homogen. residuum resoluendum		1	0	2	1	1	7	1	2
	ff				4	5	7	9	6
Rad. aucta decuplata *b* = 740	3.*bb*		1	6	4	2	8	0	0
	3.*b*					2	2	2	0
Diuisor	*B*		1	6	9	0	8	1	8
	ffc			2	7	4	7	7	6
	3.*bbc*		9	8	5	6	8	0	0
Rad. singularis tertia *c* = 6	3.*bcc*				7	9	9	2	0
	ccc						2	1	6
Ablatitium	*Bc*	1	0	2	1	1	7	1	2
Radix vniuersalis completè educta			7			4			6
Homogenei residuum finale.		0	0	0	0	0	0	0	0

E dato igitur homogeneo. 449324752. factâ ipsius ad hunc modum resolutione educta est radix 746. radici quæsititiæ *a*. æqualis, quæ ex intento problematis educenda erat.

Lemma.

Si è dato æquationis propositæ. . . *aaa* + 45796. *a* = 449324752. homogeneo radix 746. *resolutionis viâ* educta, radici quæsititiæ *a*. æqualis & æquationis explicatoria sit,

est . . . *aaa* = 746 | 746 | 746 & 45796.*a* = 45796. | 746

Sed . . . . 746 | 746 | 746 = 415160936. & 45796 | 746 = 34163816.

Et . . . + 415160936 | + . 34163816 = 449324752.

Ergo . . . . *aaa* + 45796.*a* = 449324752.

Est autem ipsa æquatio præposita.

Cougrua est igitur æquationis de radice 746. retrogradâ compositionis viâ facta explicatio, ac proinde radix eductitia 746. radici quæsititiæ *a* æqualis est, & resolutio per quam educta est radix verè facta, & canon cuius directione facta est resolutio, rite constitutus. Quod inprimis probasse oportuit.

Exemplum deuolutionis.

Æquatio resoluenda . . . $\begin{cases} aaa + ffa = ggg. \\ aaa + 95400.\,a = 1819459. \end{cases}$

Resolutionis

Resolutionis canon . . . . $\left|\;\underbrace{\begin{matrix}+ffb.\\ +bbb.\end{matrix}}_{Ab}\quad\underbrace{\begin{matrix}+.ffc & +3.bcc\\ +3.bbc & +.ccc\end{matrix}}_{Bc.}\right.$

Radix vniuersalis successiuè educenda					1			9
Homogen. resoluendum	ggg	1	8	1	9	4	5	9
	ff			9	5	4	0	0
	bb					1		
Diuisor	A			9	5	4	0	0
Rad. singularis prima $b = 1$	ffb			9	5	4	0	0
	bbb					1		
Ablatitium	Ab			9	5	5	0	0
Radix singularis					1			
Homogenei reliquum resoluendum			8	6	4	4	5	9
Radix singularis decuplata $b = 10$	ff			9	5	4	0	0
	$3.bb$					3	0	0
	$3.b$						3	0
Diuisor	B			9	5	7	3	0
	ffc		8	5	8	6	0	0
Rad. singularis secunda $c = 9$	$3.bbc$				2	7	0	0
	$3.bcc$				2	4	3	0
	ccc					7	2	9
Ablatitium . . . . 8 . .	Bc		8	6	4	4	5	9
Radix vniuersalis completè educta					1			9
Homogenei reliquum finale		0	0	0	0	0	0	0

E dato igitur homogeneo 1819459. analytice educta est radix 19. radici quæsititiæ a. æqualis, quæ educenda erat.

Exemplum rectificationis.

Æquatio resoluenda . . . . $\left\{\begin{matrix} aaa + ffa = ggg. \\ aaa + 274576.a = 301163392.\end{matrix}\right.$

Resolutionis canon . . . . $\left|\;\underbrace{\begin{matrix}+ffb\\ +bbb\end{matrix}}_{Ab}\quad\underbrace{\begin{matrix}+..ffc & +3.bcc\\ +3.bbc & +..ccc\end{matrix}}_{Bc}\right.$

Eductio

Eductio radicis singularis primæ per rectificationem.

Homogeneum datum	ggg	3 0 1 1 6 3 3 9 2
Coefficiens cubicè graduatum	—fff	—1 4 3 8 7 7 8 2 4
Differentia	ggg—fff	1 5 7 2 8 5 5 6 8
Radix singularis prima	b	5

Resolutio continuata.

Radix vniuersalis successiuè educenda		5 3 6
Homogeneum resoluendum	ggg	3 0 1 1 6 3 3 9 2
	ff	2 7 4 5 7 6
Rad. sing prima b = 5	ffb	1 3 7 2 8 8 0
	bbb	1 2 5
Ablatitium	Ab	2 6 2 2 8 8 0
Radix singularis		5
Homogenei residuum resoluendum		3 8 8 7 5 3 9 2
	ff	2 7 4 5 7 6
Radix sing. decupl. b = 50	3.bb	7 5 0 0
	3.b	1 5 0
Diuisor	B	1 0 3 9 5 7 6
	ffc	8 2 3 7 2 8
	3.bbc	2 2 5 0 0
Rad.sing.secund. c = 3	3.bcc	1 3 5 0
	ccc	2 7
Ablatitium	Bc	3 2 2 6 4 2 8
Radix aucta		5 3
Homogenei residuum resoluendum		6 7 6 1 1 1 2
	ff	2 7 4 5 7 6
Radix aucta decuplata b = 530	3.bb	8 4 2 7 0 0
	3.b	1 5 9 0
Diuisor	B	1 1 1 8 8 6 6
	ffc	1 6 4 7 4 5 6
	3.bbc	5 0 5 6 2 0 0
Radix singularis tertia c = 6	3.bcc	5 7 2 4 0
	ccc	2 1 6
Ablatitium	Bc	6 7 6 1 1 1 2
Radix vniuersalis completè educta		5 3 6
Homogenei reliquum finale		0 0 0 0 0 0

E dato

E dato igitur homogeneo. 305163392. analyticè educta est radix 536. radici quæsititiæ *a*. æqualis, quæ educenda erat.

PROBLEMA 8.

E dato æquationis . . . . . $aaa - ffa = ggg$ . . . . . in numeris propositæ homogeneo radicem radici quæsititiæ *a*. æqualem analyticè educere.

Sit æquatio numerosè proposita . . . . $aaa - 2648.\ a = 91148512.$

Ergo . . . . $2648. = ff.$ & $91148512. = ggg.$

Ponatur . . . . . . $b + c = a.$

Ergo . . . . $+ \begin{array}{l} |b+c \\ |b+c \\ |b+c \end{array} - \begin{array}{l} |ff \\ |b+c \end{array} = 91148512.$

Factis igitur homogeneis particularibus,

fit . . . . . . $\begin{array}{l} +\ ..bbb \\ +\ 3.bbc \\ +\ 3.bcc \\ +\ ..ccc \end{array} \quad \begin{array}{l} -ffb \\ -ffc \end{array} = 91148512.$

Et iisdem bifariam ad hunc modum distributis,

fit . . . . . . $\underbrace{\begin{array}{l} -ffb \\ +bbb \end{array}}_{Ab} \quad \underbrace{\begin{array}{l} -\ ..ffc + 3.bcc \\ +\ 3.bbc + ..ccc \end{array}}_{Bc} = 91148512.$

Est autem æquationis huius pars speciosa bipartita *Ab*. *Bc*. resolutionis canon cuius applicatione operis analytici processus dirigendus est, quem ritè constitutum esse ex sequenti Lemmate constabit.

Facta igitur canonis huius applicatione omnino vt in subiecti exempli schematismo ordinata conspicitur, fiat ipsius directione dati homogenei 91148512. ad radicem ex eo educendam resolutio, vt sequitur.

Radix vniuersalis successiuè educenda			4			5			2
Homogeneum resoluendum		9	1	1	4	8	5	1	2
	$-ff$			2	6	4	8		
	$3.bb$	1	6						
Diuisor	A	1	5	7	3	5	2		
	ffb	—	1	0	5	9	2		
Radix singula. prima $b = 4$	bbb	6	4						
Ablatitium	$Ab.$	6	2	9	4	0	8		
Radix singularis			4						
Homogenei residuum resoluendum		2	8	2	0	7	7	1	2

Radix singularis		4
Homogenei residuum resoluendum		2 8 2 0 7 7 1 2
	$-ff$	2 6 4 8
Radix singularis decupl. $b = 40$	$3.bb$	4 8 0 0
	$3.b$	1 2 0
	$+$	4 9 2 0
Diuisor	B	4 8 9 3 5 2
	$-ffc$	1 3 2 4 0
	$3.bbc$	2 4 0 0 0
Rad. singul. secunda $c = 5$	$3.bcc$	3 0 0 0
	ccc	1 2 5
	$+$	2 7 1 2 5
Ablatitium	Bc	2 6 9 9 2 6 0
Radix aucta		4 5
Homogenei reliquum resoluendum		1 2 1 5 1 1 2
	$-ff$	2 6 4 8
Rad. aucta decuplata $b = 450$	$3.bb$	6 0 7 5 0 0
	$3.b$	1 3 5 0
	$+$	6 0 8 8 5 0
Diuisor	B	6 0 6 2 0 2
	$-ffc$	5 2 9 6
	$3.bbc$	1 2 1 5 0 0 0
Rad. singularis tertia $c = 2$	$3.bcc$	5 4 0 0
	ccc	8
	$+$	1 2 2 0 4 0 8
Ablatitium	Bc	1 2 1 5 1 1 2
Radix vniuersalis completè educta		4 5 2
Homogenei residuum finalè.		0 0 0 0 0 0 0 0

E dato igitur homogeneo 91148512. factâ ipsius ad hunc modum resolutione educta est radix 452. radici quæsititiæ a. æqualis, quæ ex intento Problematis educenda erat.

Lemma.

Si è dato propositæ æquationis . . . . $aaa - 2648.a = 91148512$. homogeneo radix 452. resolutionis viâ educta, radici quæsititiæ a. æqualis & æquationis explicatoria sit,

est . . . $aaa = \begin{array}{|l}452\\452\\452\end{array}$ & $2648.a = \begin{array}{|l}2468.\\452\end{array}$

Sed

Sed . . . . $\begin{array}{l|} 452 \\ 452 \\ \underline{452} \end{array}$ ═══ 92345408. & $\begin{array}{l|} 2468 \\ \underline{452} \end{array}$ ═══ 1196896.

Et . . . . $\begin{array}{l|} + 92345408 \\ - .1196896 \end{array}$ ═══ 91148512.

Ergo . . . . $aaa - 2468.a$ ═══ 91148512.

Est autem ipsa æquatio proposita.

Congrua est igitur æquationis propositæ de radice 452. retrogradâ compositionis viâ facta explicatio, ac proinde radix eductitia 452. radici quæsititiæ a æqualis est, & resolutio per quam educta est radix verè facta, & canon cuius directione facta est resolutio, ritè constitutus. Quod inprimis probasse oportuit.

Nota

Notandum hîc est, communia affectionis signa + & — quæ in lineari canonis seriè cum reliquis notis coordinata in æquationibus negatiuè affectis habentur, ad particularium proximè præcedentium Summas, vbi plura eiusdem affectionis (siue alterius siue vtriusque) occurrunt, separatim significandas, inserta esse: idque eo fine, vt affirmatorum & negatorum differentiæ totales, pro diuisoribus atque ablatitijs constituendis, distinctè appareant. Vide Notam 2. ad Problema primum.

Casus Anticipationis.

In æquatione $aaa - ffa$ ═══ ggg, numerosè propositâ accidit interdum vt coefficiens diuisorium pluribus abundet binis figuris quam homogeneum resoluendum ternis. Itaq; vt resolutioni sit locus præponatur homogeneo ad leuam ea cyphrarum multitudo vt illud tot puncta cubica recipiat quot coefficiens quadratica: Et ad primum punctum vacuum tanquam per anticipationem opus resolutionis inchoetur. In quo hoc inest compendij vt prima radix quadratica è coefficiente educta, primæ radici singulari è dato homogeneo educendæ aut æqualis sit aut eâ proximè minor.

Anticipationis exemplum.

Æquatio resoluenda . . . $\left\{ \begin{array}{l} aaa - ffa ═══ ggg. \\ aaa - 116620 ═══ 352947. \end{array} \right.$

Resolutionis canon . . . . $\left| \begin{array}{l} \underbrace{\begin{array}{l} -ffb. \\ +bbb. \end{array}}_{Ab} \quad \underbrace{\begin{array}{ll} -.ffc & +3bcc \\ +3.bbc & +.ccc \end{array}}_{Bc.} \end{array} \right.$

Radix vniuersalis successiuè educenda		3			4			3
Homogen. resoluendum	ggg .	0	3	5	2	9	4	7

Radix

Radix vniuersalis succeßiuè educenda		3 . . 4 . . 3
Homogeneum resoluendum	ggg . . .	0 3 5 2 9 4 7
	—ff . .	—1 1 6 6 2 0
	bb	9
Diuisor	—A . .	—2 6 6 2 0
	—ffb . .	—3 4 9 8 6 0
Rad. singularis prima $b = 1$	bbb . . .	2 7
Ablatitium	—Ab . . .	—7 9 8 6 0
Radix singularis		3
Homogenei reliquum resoluendum		8 3 3 8 9 4 7
	—ff . .	—1 1 6 6 2 0
Radix singularis decuplata $b = 30$	3.bb	2 7 0 0
	3.b	9 0
	+	2 7 9 0
Diuisor	B	1 6 2 3 8 0
	—ffc . .	— 4 6 6 4 8 0
	3.bbc . .	1 0 8 0 0
Rad. singularis secunda $c = 4$	3.bcc	1 4 4 0
	ccc	6 4
	+	1 2 3 0 4
Ablatitium	Bc	7 6 3 9 2 0
Radix aucta		3 . . 4
Homogen. residuum resoluendum		6 9 9 7 4 7
	—ff . . .	—1 1 6 6 2 0
Radix aucta decuplata $b = 340$	3.bb	3 4 6 8 0 0
	3.b	1 0 2 0
	+	3 4 7 8 2 0
Diuisor	B	2 3 1 2 0 0
	—ffc . . .	—3 4 9 8 6 0
	3.bbc . . .	1 0 4 0 4 0 0
Radix singularis tertia $c = 3$	3.bcc	9 1 8 0
	ccc	2 7
	+	1 0 4 9 6 0 7
Ablatitium	Bc	6 9 9 7 4 7
Radix vniuersalis completè educta		3 . . 4 . . 3
Homogenei reliquum finale		0 0 0 0 0 0

E dato

E dato igitur homogeneo 352947. analyticè educta est radix 343. radici quæsititiæ *a.* æqualis, quæ educenda erat.

Rectificationis Exemplum.

Æquatio resoluenda . . . . $\begin{cases} aaa - ffa = ggg. \\ aaa - 127296.a = 85700000. \end{cases}$

Resolutionis canon . . . . $\left| \begin{array}{ll} \underbrace{\begin{array}{l} -ffb \\ +bbb \end{array}}_{Ab} & \underbrace{\begin{array}{ll} -..ffc & +3.bcc \\ +3.bbc & +..ccc \end{array}}_{Bc} \end{array} \right.$

Eductio radicis singularis primæ per rectificationem.

Homogeneum datum	ggg	8 5 7 6 0 0 0 0
Coefficiens cubicè graduatum	fff	4 5 4 4 4 6 7 2
Summa	ggg+fff	1 3 1 2 0 4 6 7 2
Radix singularis prima	b.	5

Resolutio continuata.

Radix vniuersalis sucessiuè educenda		5 3 6
Homogeneum resoluendum	ggg	8 5 7 6 0 0 0 0
	−ff	−1 2 7 2 9 6
Rad. singularis prima b = 5	−ffb	−6 3 6 4 8 0
	bbb	1 2 5
Ablatitium	−Ab	6 1 3 5 2 0
Radix singularis		5
Homogenei reliquum resoluendum		2 4 4 0 8 0 0 0
	−ff	−1 2 7 2 9 6
Radix singularis decuplata b = 50	3.bb	7 5 0 0
	3.b	1 5 0
	+	7 6 5 0
Diuisor	B	6 3 7 7 0 4
	−ffc	3 8 1 8 8 8
	3.bbc	2 2 5 0 0
Rad. singularis secunda c = 3	3.bcc	1 3 5 0
	ccc	2 7
	+	1 3 7 7
Ablatitium	Bc	2 0 0 5 8 1 2
Radix aucta		5 3
Hmogenei residuum resoluendum		4 3 4 9 8 8 8

Radix aucta		5			3			
Homogen. residuum resoluendum		4	3	4	9	8	8	8
	$-ff$	—	1	2	7	2	9	6
Radix aucta decuplata $b = 530$	$3.bb$		8	4	2	7	0	0
	$3.b$				1	5	9	0
	$+$		8	4	4	2	9	0
Diuisor	B		7	1	6	6	9	4
	$-ffc$	—	7	6	3	6	7	6
	$3.bbc$	5	0	5	6	2	0	0
Radix singularis tertia $c = 6$	$3.bcc$			5	7	2	4	0
	ccc					2	1	6
	$+$	5	1	1	3	6	5	6
Ablatitium	Bc	4	2	4	9	8	8	8
Radix vniuersalis complete educta		5			3			6
Homogenei reliquum finale			0	0	0	0	0	0

E dato igitur homogeneo. 85760000. analyticè educta est radix 536. radici quæsititiæ *a.* æqualis, quæ educenda erat.

PROBLEMA 9.

E dato æquationis . . . $-aaa + ffa = ggg$ . . . quæ de duabus radicibus explicabilis est, in numeris propositæ homogeneo radicem vtramque radici ambiguæ *a.* æqualem analyticè educere.

Sit æquatio numerose proposita . . . . $-aaa + 52416.\ a = 1244160.$

Ergo . . . . $52416 = ff.$ & $1244160. = ggg.$

Ponatur . . . . . $b + c = a.$

Ergo . . . . $-\left|\begin{matrix} b+c \\ b+c \\ b+c \end{matrix}\right. + \left|\begin{matrix} ff \\ \underline{b+c} \end{matrix}\right. = 1244160.$

Factis igitur homogeneis particularibus,

fit . . . . . . $\begin{matrix} -..bbb \\ -3.bbc \\ -3.bcc \\ -..ccc \end{matrix}$ $\begin{matrix} +ffb \\ +ffc \end{matrix} = 1244160.$

Et iisdem bifariam ad hunc modum distributis,

fit . . . . . . $\underbrace{\begin{matrix} +ffb \\ -bbb \end{matrix}}_{Ab}$ $\underbrace{\begin{matrix} +..ffc - 3.bcc \\ -3.bbc - ..ccc \end{matrix}}_{Bc} = 1244160.$

Est

Est autem æquationis huius pars speciosa bipartita *Ab. Bc.* resolutionis canon cuius applicatione operis analytici processus dirigendus est, quem ritè constitutum esse per applicationis congruentiam superioris Lemmatis exemplo demonstrari potest.

Facta igitur canonis huius applicatione omnino vt in subiecto exempli schemate ordinata conspicitur, fiat ipsius directione dati homogenei 1244160. ad vtramque radicem ex eo educendam resolutio, vt sequitur.

Eductio radicis maioris.

Radix vniuersalis successiuè educenda		2 4 1 6
Homogeneum resoluendum	*SSS*	1 2 4 4 1 6 0
	ff	5 2 4 1 6
	—3.*bb*	—4
Diuisor	*A*	1 2 4 1 6
	ffb	1 0 4 8 3 2
Radix singula. prima *b* = 4	—*bbb*	—8
Ablatitium	—*Ab.*	—2 4 8 3 2
Radix singularis		2
Homogenei residuum resoluendum		—1 2 3 9 0 4 0
	ff	5 2 4 1 6
Radix singularis decupl. *b* = 20	—3.*bb*	—1 2 0 0
	3.*b*	—6 0
	—	—1 2 6 0
Diuisor	—*B*	—7 3 5 8 4
	ffc	5 2 4 1 6
	—3.*bbc*	—1 2 0 0
Rad. singul. secunda *c* = 1	—3.*bcc*	—6 0
	—*ccc*	—1
	—	—1 2 6 1
Ablatitium	—*Bc*	—7 3 6 8 4
Radix aucta		2 1
Homogenei reliquum resoluendum		—5 0 2 2 0 0
	ff	5 2 4 1 6
Rad. aucta decuplata *b* = 450	—3.*bb*	—1 3 2 3 0 0
	—3.*b*	—6 3 0
	—	—1 3 2 9 3 0
Diuisor	—*B*	—8 0 5 1 4
	ffc	3 1 4 4 9 6
	—3.*bbc*	—7 9 3 8 0 0
Rad. singularis tertia *c* = 2	—3.*bcc*	—2 2 6 8 0
	—*ccc*	—2 1 6
	—	—8 1 6 0 9 6
Ablatitium	—*Bc*	—5 0 2 2 0 0
Radix vniuersalis completè educta		2 4
Homogenei reliquum finale		0 0 0 0 0 0 0

Eductio

Eductio radicis minoris factâ deuolutione.

Radix vniuersalis		2 . . 4
Homogeneum resoluendum		1 2 4 4 1 6 0
	ff	5 2 4 1 6
	$-bb$	4
Diuisor	A	5 2 0 1 6
	ffb	1 0 4 8 3 2
Radix singularis prima $b = 2$	$-bbb$	8
Ablatitium	Ab	1 0 4 8 3 2
Radix singularis		2
Homogenei residuum resoluendum		2 0 3 8 4 0
	ff	5 2 4 1 6
Radix sing. decupl. $b = 20$	$-3.bb$	1 2 0 0
	$-3.b$	6 0
	$-$	1 2 6 0
Diuisor	B	5 1 1 5 6
	ffc	2 0 9 6 6 4
	$-3.bbc$	4 8 0 0
Rad. sing. secund. $c = 4$	$-3.bcc$	9 6 0
	$-ccc$	6 4
	$-$	5 8 2 4
Ablatitium	Bc	2 0 3 8 4 0
Radix vniuersalis completè educta		2 . . 4
Homogen. residuum finale.		0 0 0 0 0 0

E dato igitur homogeneo 1244160. factâ ipsius ad hunc modum resolutione eductæ sunt duæ radices 216. & 24. radici quæsititiæ 4. æqualis vtraque, quæ ex intento Problematis educendę erant.

Compendium.

Si propositæ æquationis $-aaa + ffa = ggg$. radices ponantur b. & c. b. maior. c. vero minor, erit (per propos. 6. Sect. 4.) coefficiens $ff = bb + bc + cc$.

Ergo si data sit radix b. maior, erit æquatio $cc + bc = ff - cc$. cuius radix quæsititia c. minor.

Vel si data sit c. minor, erit æquatio $bb + cb = ff - cc$. cuius radix quæsititia b. maior.

Ergo propositæ æquatioins cubicæ inuenta radice vna, per æquationis quadraticæ analysin exhibetur altera, quod pro compendio esse potest.

PRO-

PROBLEMA 10.

E dato æquationis . . . $aaa - daa = ggg$ . . . in numeris propositæ homogeneo radicem radicis quæsititiæ a. valorem analyticè educere.

Sit æquatio numerosè proposita . . . . $aaa - 68.\ aa = 134454528$.

Ergo . . . . $68 = d$. & $134454528. = ggg$.

Ponatur . . . . . $b + c = a$.

Ergo . . . . $+ \begin{array}{|l} b+c \\ b+c \\ b+c \end{array} - \begin{array}{|l} d \\ b+c \\ b+c \end{array} = 134454528.$

Factis igitur homogeneis particularibus,

fit . . . . . . $\begin{array}{ll} +..bbb & -dbb \\ +3.bbc & -2.dbc \\ +3.bcc & -.dcc \\ +..ccc & \end{array} = 134454528.$

Et ijsdem bifariàm ad hunc modum distributis,

fit . . . . / . $\underbrace{\begin{array}{l} -dbb \\ +bbb \end{array}}_{Ab} \quad \underbrace{\begin{array}{l} -..dcc+3.bbc \\ -2.dbc+3.bcc+ccc \end{array}}_{Bc} = 134454528.$

Est autem æquationis huius pars speciosa bipartita Ab. Bc. resolutionis canon, cuius applicatione operis analytici processus dirigendus est. Quem ritè constitutum esse ex sequenti Lemmate constabit.

Factâ igitur canonis huius applicatione, omninò vt in subiecto exempli schematismo ordinata conspicitur, fiat ipsius directione dati homogenei 134454528. ad radicem ex eo educendam resolutio, vt sequitur.

Radix vniuersalis successiuè educenda				5			3			6
Homogeneum resoluendum	ggg .	1	3	4	4	5	4	5	2	8
	$-d$				−6	8				
	bb		2	5						
Diuisor	A . . .		.2	4	3	2				
	$-dbb$ . .		−1	7	0	0				
Radix singula. prima $b = 5$	bbb .	1	2	5						
Ablatitium	Ab. . .	1	0	8	0	0				
Radix singularis				5						
Homogenei residuum resoluendum			2	6	4	5	4	5	2	8

Radix singularis		5
Homogenei residuum resoluendum		2 6 4 5 4 5 2 8
	— *d*	6 8
	— 2. *db*	6 8 0 0
Radix singularis decupl. *b* ═ 50	3. *bb*	7 5 0 0
	3. *b*	1 5 0
	+	7 6 5 0
	—	6 8 6 8
Diuisor	*B*	6 9 6 3 2
	— *dcc*	6 1 2
	— 2. *dbc*	2 0 4 0 0
Rad. singul. secunda *c* ═ 3	3. *bbc*	2 2 5 0 0
	3. *bcc*	1 4 5 0
	ccc	2 7
	+	2 3 8 7 7
	—	2 1 0 1 2
Ablatitium	*Bc*	2 1 7 7 5 8
Radix aucta		5 3
Homogenei residuum resoluendum		4 6 7 8 7 2 8
	— *d*	6 8
	— 2. *db*	7 2 0 8 0
Rad. aucta decuplata *b* ═ 530	3. *bb*	8 4 2 7 0 0
	3. *b*	1 5 9 0
	+	8 4 4 2 9 0
	—	7 2 1 4 8
Diuisor	*B*	7 7 2 1 4 2
	— *dc*	2 4 4 8
	— 2. *dbc*	4 3 2 4 0 0
Rad. singularis tertia *c* ═ 6	3. *bbc*	5 0 5 6 2 0 0
	3. *bcc*	5 7 2 4 0
	ccc	2 1 6
	+	5 1 1 3 6 5 6
	—	4 3 4 8 4 8
Ablatitium	*Bc*	4 6 7 8 7 2 8
Radix vniuersalis completè educta		5 3 6
Homogenei reliquum finale		0 0 0 0 0 0 0 0

E dato igitur homogeneo 134454528. factâ ipsius ad hunc modum resolutione educta est radix 536. radici quæsititiæ *a* æqualis, quæ ex intento Problematis educenda erat.

Lemma.

Lemma.

Si è dato propositæ æquationis . . . . $aaa - 68.\,a = 134454528.$ homogeneo radix 536. resolutionis viâ educta, radici quæsititiæ a. æqualis & æquationis explicatoria sit,

$$\text{est} \ldots aaa = \left|\begin{array}{l}536\\536\\536\end{array}\right. \quad \& \quad 68.\,aa = \left|\begin{array}{l}68.\\536\\536\end{array}\right.$$

$$\text{Sed} \ldots \left.\begin{array}{r}536\\536\\536\end{array}\right| = 153990656. \quad \& \quad \left.\begin{array}{r}68\\536\\536\end{array}\right| = 19536128.$$

$$\text{Atq;} \ldots \left.\begin{array}{r}+153990656\\-\;.\;19536128\end{array}\right| = 134454528.$$

$$\text{Ergo} \ldots aaa - 68.\,aa = 134454528.$$

Est autem ipsa æquatio proposita.

Congrua est igitur æquationis propositæ de radice 536. retrogradâ compositionis viâ facta explicatio, ac proinde radix eductitia 536. radici quæsititiæ a æqualis est, & resolutio per quam educta est radix verè facta, & canon cuius directione facta est resolutio, ritè constitutus. Quod inprimis probasse oportuit.

Ad æquationes biquadraticas resoluendas.

PROBLEMA II.

E dato æquationis biquadraticæ simplicis . . . . $aaaa = bbbb$ . . numerosè propositæ homogeneo radicem radici quæsititiæ a. æqualem analyticè educere.

Sit æquatio numerosè proposita $aaaa = 19565295376.$

Et ponatur . . . . $b + c = a.$

$$\text{Ergo} \ldots \left|\begin{array}{l}b+c\\b+c\\b+c\\b+c\end{array}\right| = 19565295376.$$

Factis igitur homogeneis particularibus & bifariam, vt oportet distributis,

$$\text{fit} \ldots \underbrace{+\,bbbb}_{Ab} \quad \underbrace{\begin{array}{l}+\,4.\,bbbc\\+\,6.\,bbcc\\+\,4.\,bccc\\+\;..\,cccc\end{array}}_{Bc} = 19565295376.$$

Est

Est autem æquationis huius pars speciosa bipartita *Ab. Bc.* resolutionis canon, cuius applicatione operis analytici processus dirigendus est. Quem ritè constitutum esse superiorum Lemmatum exemplo manifestari potest.

Factâ igitur canonis huius applicatione omninò vt in subiecto exempli schematismo ordinata conspicitur, fiat ipsius directione dati homogenei 19565295376. ad radicem ex eo educendam resolutio, vt sequitur.

Radix vniuersalis successiuè educenda		3 . . . 7 . . . 4
Homogeneum resoluendum	*bbbb*..	1 9 5 6 5 2 9 5 3 7 6
Diuisor	*bbb*. ..	2 7
Rad. sing. prima $b = 3$		
Ablatitium.	*bbbb* ..	8 1
Radix singularis		3
Homogenei reliquum resoluendum		1 1 4 6 5 2 9 5 3 7 6
	4 *bbb* ..	1 0 8 0 0 0
Radix sing. decup. $b = 30$	6. *bb*	5 4 0 0
	4. *b*	1 2 0
Diuisor	*B* ..	1 1 3 5 2 0
	4 *bbbc* ..	7 5 6 0 0 0
	6. *bbcc* ..	2 6 4 6 0 0
Rad. singularis secunda $c = 7$	4. *bccc* ...	4 1 1 6 0
	cccc	2 4 0 1
Ablatitium	*Bc* ..	1 0 6 4 1 6 1
Radix aucta		3 . . . 7
Homogenei residuum resoluendum		8 2 3 6 8 5 3 7 6
	4. *bbb* ..	2 0 2 6 1 2 0 0 0
Radix aucta decup. $b = 370$	6. *bb*	8 2 1 4 0 0
	4. *b*	1 4 8 0
Diuisor	*B*	2 0 3 4 3 4 8 8 0
	4 *bbbc* ...	8 1 0 4 4 8 0 0 0
	6 *bbcc* ...	1 3 1 4 2 4 0 0
Radix singularis tertia $c = 4$	4. *bccc*	9 4 7 2 0
	cccc	2 5 6
Ablatitium	*Bc* ...	8 2 3 6 8 5 3 7 6
Radix vniuersalis completè educta		3 . . . 7 . . . 4
Homogenei residuum finale		0 0 0 0 0 0 0 0 0

E dato igitur homogeneo 19565295376. factâ ipsius ad hunc modum resolutione educta est radix 374. radici quæsititiæ *a.* æqualis, quæ ex intento Problematis educenda erat.

PRO-

PROBLEMA 12.

E dato æquationis . . . . $aaaa - ggg\,a = hhhh$ . . in numeris propoſitæ homogeneo radicem radicis quæſititiæ a. valorem analyticè educere.

Sit æquatio numeroſè propoſita . . . . $aaaa - 426.\,a = 2068948.$

Ergo . . . . $426 = ggg$. & $2068948 = hhhh$.

Ponatur . . . . . $b + c = a$.

Ergo . . . . $+\left|\begin{array}{l} b+c \\ b+c \\ b+c \\ b+c \end{array}\right. - ggg\,|\,b+c = 2068948.$

Factis igitur homogeneis particularibus,

fit . . . . . . $\begin{array}{l} +..bbbb \\ +4.bbbc \\ +6.bbcc \\ +4.bccc \\ +.cccc \end{array} \begin{array}{l} -gggb \\ -gggc \\ \\ \\ \\ \end{array} = 206848.$

Et ijſdem bifariàm ad hunc modum diſtributis,

fit . . . . . . $\underbrace{\begin{array}{l} -gggb \\ +bbbb \end{array}}_{Ab} \quad \underbrace{\begin{array}{l} -..gggc + 6.bbcc \\ +4.bbbc + 4.bccc + cccc \end{array}}_{Bc} \ldots = 2068948.$

Eſt autem æquationis huius pars ſpecioſa bipartita $Ab.\ Bc.$ reſolutionis canon, cuius applicatione operis analytici proceſſus dirigendus eſt. Quem ritè conſtitutum eſſe ſuperiorum Lemmatum exemplo manifeſtari poteſt.

Factâ igitur canonis huius applicatione omninò vt in ſubiecto exempli ſchematiſmo ordinata conſpicitur, fiat ipſius directione dati homogenei 2068948. ad radicem ex eo educendam reſolutio, vt ſequitur.

Radix vniuerſalis ſucceſſiuè educenda				3				8
Homogeneum reſoluendum	$-ggg$ . . .	2	0	6	8	9	4	8
	$-ggg$				4	2	6	
	bbb		2	7				
Diuiſor	A		2	6	5	7	4	
	$-gggb$			−1	2	7	8	
Radix ſingula. prima $b = 5$	$bbbb$		8	1				
Ablatitium	Ab.		7	9	7	2	2	
Radix ſingularis					3			
Homogenei reſiduum reſoluendum		1	2	7	1	7	2	8

 Radix

Radix singularis		3
Homogenei reliquum resoluendum		1 2 7 1 7 2 8
	—ggg	4 2 6
	4.bbb	1 0 8 0 0 0
Radix sing. decup. $b = 30$	6.bb	5 4 0 0
	4.b	1 2 0
	+	1 1 3 5 2 0
Diuisor	B	1 1 3 0 9 4
	—gggc	3 4 0 8
	4.bbbc	8 6 4 0 0 0
Rad. singularis secunda $c = 8$	6.bbcc	3 4 5 6 0 0
	4 bccc	6 1 4 4 0
	cccc	4 0 9 6
	+	1 2 7 5 1 3 6
Ablatitium	Bc	1 2 7 1 7 2 8
Radix vniuersalis completè educta		3 8
Homogenei residuum finale		0 0 0 0 0 0 0

E dato igitur homogeneo 2068948. factâ ipsius ad hunc modum resolutione educta est radix 38. radici quæsititiæ *a.* æqualis, quæ ex intento Problematis educenda erat.

Anticipationis exemplum.

Æquatio resoluenda . . . . $\left\{ \begin{array}{l} aaaa - ggga = hhhh. \\ aaaa - 43602354.a = 4172608. \end{array} \right.$

Resolutionis canon . . . . $\left| \begin{array}{ll} \underbrace{\begin{array}{l} -gggb \\ +bbbb \end{array}}_{Ab} & \underbrace{\begin{array}{llll} -..gggc & +6.bbcc. & & \\ +4bbbb & +4.bccc & +..cccc & \end{array}}_{Bc} \end{array} \right.$

Radix vniuersalis successiuè educenda		3 5 2
Homogeneum resoluendum	hhhh .	0 0 4 1 7 2 6 0 8
	—ggg .	—4 3 6 0 2 3 5 4
	bbb .	2 7
Diuisor	—B .	—1 6 6 0 2 3 5 4
	—gggb	—1 3 0 8 0 7 0 6 2
Rad. sing. prima $b = 3$	bbbb .	8 1
Ablatitium	—Ab .	—4 9 8 0 7 0 6 2
Radix singularis		3
Homogenei residuum resoluendum		4 9 8 4 8 7 8 8 0 8

Radix

Radix singularis		3
Homogenei residuum resoluendum		4 9 8 4 8 7 8 8 0 8
	$-ggg$	— 4 3 6 0 2 3 5 4
	$4.bbb$	1 0 8 0 0 0
Radix singularis decupl. $b = 30$	$6.bb$	5 4 0 0
	$4.b$	1 2 0
	+	1 1 3 5 2 0
Diuisor	B	6 9 9 1 7 6 4 6
	$-gggc$	—2. 1 8 0 1 1 7 7 0
	$4bbbc$	5 4 0 0 0 0
Rad. singul. secunda $c = 5$	$6.bbcc$	1 3 5 0 0 0
	$4.bccc$	1 5 0 0 0
	$cccc$	6 2 5
	+	6 9 0 6 2 5
Ablatitium	Bc	4 7 2 6 1 3 2 3 0
Radix aucta		3 5
Homogenei residuum resoluendum		2 5 8 7 4 6 5 0 8
	$-ggg$	— 4 3 6 0 2 3 5 4
	$4.bbb$	1 7 1 5 0 0 0 0 0
Rad. aucta decuplata $b = 350$	$6.bb$	7 3 5 0 0 0
	$4.b$	1 4 0 0
	+	1 7 2 2 3 6 4 0 0
Diuisor	B	1 2 8 5 3 4 0 4 6
	$-gggc$	— 8 7 2 0 4 7 0 8
	$4.bbbc$	3 4 3 0 0 0 0 0 0
Rad. singularis tertia $c = 2$	$6.bbcc$	2 9 4 0 0 0 0
	$4.bccc$	1 1 2 0 0
	$cccc$	1 6
	+	3 4 5 8 5 1 2 1 6
Ablatitium	Bc	2 5 8 7 4 6 5 0 8
Radix vniuersalis completè educta		3 5 2
Homogenei reliquum finale		0 0 0 0 0 0 0 0 0 0

E dato igitur homogeneo 4172608. factâ ipsius ad hunc modum resolutione educta est radix 352. radici quæsititiæ a æqualis, quæ educenda erat.

PROBLEMA 13.

E dato æquationis . . . $aaaa - ffaa + ggga = hbbb$. . . in numeris propositæ homogeneo radicem radicis quæsititiæ a. valorem analyticè educere.

Sit

Sit æquatio numerosè proposita $aaaa - 1024.aa + 6254.a = 19633735875$.

Ergo . . . $1024 = ff$. & $6254 = ggg$. & $19633735875 = bbbb$.

Ponatur . . . . $b + c = a$.

Ergo . . . . . . $+ (b+c)^4 - ff(b+c)^2 + ggg(b+c) = 19633735875$.

Factis igitur homogeneis particularibus,

fit . . . . $+ bbbb - ffbb + gggb$
$+ 4.bbbc - 2.ffbc + gggc$
$+ 6.bbcc - ffcc$
$+ 4.bccc$
$+ cccc = 19633735875$.

Et ijsdem bifariam ad hunc modum etdistributis,

fit . . . . $\underbrace{+ gggb - ffbb + bbbb}_{Ab} \; \underbrace{+ gggc - ffcc + 4.bbbc + 6.bbcc - 2.ffbc + 4.bccc + cccc}_{Bc} = 19633735875$.

Est autem æquationis huius pars speciosa bipartita Ab. Bc. resolutionis canon, cuius applicatione operis analytici processus dirigendus est. Quem ritè constitutum esse superiorum Lemmatum exemplo demonstrari potest.

Factâ igitur canonis huius applicatione, omninò vt in subiecto exempli schematismo ordinata conspicitur, fiat ipsius directione dati homogenei 19633735875. ad radicem ex eo educendam resolutio, vt sequitur.

Radix vniuersalis successiuè educenda				3				7				5
Homogeneum resoluendum	$bbbb$	1	9	6	3	3	7	3	5	8	7	5
	ggg							6	2	5	4	
	$-ff$				—	1	0	2	4			
	bbb		2	7								
	$+$		2	7	0	0	6	2	5	4		
Diuisor	A		2	6	9	0	3	8	5	4		
	$gggb$					1	8	7	6	2		
Radix sing. prima $b = 3$	$-ffbb$			—	9	2	1	6				
	$bbbb$	8	1									
	$+$	8	1	0	1	8	7	6	2			
Ablatitium	Ab	8	0	0	9	7	1	6	2			
Radix singularis				3								
Homogen. residuum resoluendum		1	1	6	2	4	0	1	9	6	7	5

Radix

Radix singularis		3
Homogenei residuum resoluendum		1 1 6 2 4 0 1 9 6 7 5
	ggg	6 2 5 4
	$-ff$	1 0 2 4
	$-2.ffb$	6 1 4 4 0
Radix sing. decupl. $b = 30$	$4.bbb$	1 0 8 0 0 0 0
	$6.bb$	5 4 0 0
	$4.b$	1 2 0
	$+$	1 1 3 5 2 6 2 5 4
	$-$	6 2 4 6 4
Diuisor	B	1 1 2 9 0 1 6 1 4
	$gggc$	4 3 7 7 8
	$-ffcc$	5 0 1 7 6
	$-2\,ffbc$	4 3 0 0 8 0
Rad. sing. secund. $c = 7$	$4.bbbc$	7 5 6 0 0 0 0
	$6.bbcc$	2 6 4 6 0 0
	$4.bccc$	4 1 1 6 0
	$cccc$	2 4 0 1
	$+$	1 0 6 4 2 0 4 7 7 6
	$-$	4 8 0 2 5 6
Ablatitium	Bc	1 0 5 9 4 0 2 2 1 8
Radix aucta		3 7
Homogenei residuum resoluendum		1 0 2 9 9 9 7 4 9 5
	ggg	6 2 5 4
	$-ff$	1 0 2 4
	$-2.ffb$	7 5 7 7 6 0
Rad. aucta decuplata $b = 370$	$4.bbb$	2 0 2 6 1 2 0 0 0
	$6.bb$	8 2 1 4 0 0
	$4.b$	1 4 8 0
	$+$	2 0 3 4 4 1 1 3 8
	$-$	7 5 8 7 8 4
Diuisor	B	2 0 2 6 8 2 3 5 4
	$gggc$	3 1 2 7 0
	$-ffcc$	2 5 6 0
	$-2.ffbc$	3 7 8 8 8 0 0
Rad. singularis tertia $c = 5$	$4.bbbc$	1 0 1 3 0 6 0 0 0 0
	$6.bbcc$	2 0 5 3 5 0 0 0
	$4.bccc$	1 8 5 0 0 0
	$cccc$	6 2 5
	$+$	1 0 3 3 8 1 1 8 9 5
	$-$	3 8 1 4 4 0 0
Ablatitium	Bc	1 0 2 9 9 9 7 4 9 5
Radix vniuersalis completè educta		3 7 5
Homogenei reliquum finale		0 0 0 0 0 0 0 0 0 0

E dato igitur homogeneo 19633735875. factâ ipsius ad hunc modum resolutione educta est radix 375. radici, quæsititiæ a æqualis, quæ ex intento Problematis educenda erat.

PROBLEMA 14.

E dato æquationis . . . $aaaa - ffaa - ggga = bbbb$. . . in numeris propositæ homogeneo radicem radicis quæsititiæ a. valorem analyticè educere.

Sit æquatio numerosè proposita . . $aaaa - 1024.aa - 6254.\ a = 19629045375$

Ergo . . . $1024 = ff$. & $6254 = ggg$. & $19629045375 = bbbb$.

Ponatur . . . . $b+c = a$.

Ergo . . . .
$$+\left|\begin{array}{l} b+c \\ b+c \\ b+c \\ b+c \end{array}\right. - \left|\begin{array}{l} ff \\ b+c \\ b+c \end{array}\right. - \left|\begin{array}{l} ggg \\ b+c \end{array}\right. = 19629045375.$$

Factis igitur homogeneis particularibus,

fit . . . .
$$\begin{array}{lll} +\ bbbb & -\ ffbb & -\ gggb \\ +\ 4.\ bbbc & -\ 2.\ ffbc & -\ gggc \\ +\ 6.\ bbcc & -\ ffcc & \\ +\ 4.\ bccc & & \\ +\ cccc & & \end{array} = 19629045375.$$

Et ijsdem bifariam ad hunc modum distributis,

fit . . . .
$$\underbrace{\begin{array}{l} -\ gggb \\ -\ ffbb \\ +\ bbbb \end{array}}_{Ab}\ \underbrace{\begin{array}{llll} -\ gggc & +\ 4.\ bbbc & & \\ -\ ffcc & +\ 6.\ bbcc & & \\ -\ 2\ ffbc & +\ 4\ bccc & +\ cccc & \end{array}}_{Bc} = 19629045375.$$

Est autem æquationis huius pars speciosa bipartita $Ab.\ Bc.$ resolutionis canon, cuius applicatione operis analytici processus dirigendus est. Quem ritè constitutum esse superiorum Lemmatum exemplo demonstrari potest.

Factâ igitur canonis huius applicatione, omninò vt in subiecto exempli schematismo ordinata conspicitur, fiat ipsius directione dati homogenei 19629045375. ad radicem ex eo educendam resolutio, vt sequitur.

Radix vniuersalis			3				7				5
Homogeneum resoluendum	1	9	6	2	9	0	4	5	3	7	5

Radix

Radix vniuersalis successiuè educenda		3 · 7 · 5
Homogeneum resoluendum	bbbb	1 9 6 2 9 0 4 5 3 7 5
	— ggg	6 2 5 4
	— ff	1 0 2 4
	bbb	2 7
	—	1 0 8 6 5 4
Diuisor	A	2 6 8 9 1 3 4 6
Radix sing. prima b = 3	— gggb	1 8 7 6 2
	— ffbb	9 2 1 6
	bbbb	8 1
	—	9 4 0 3 6 2
Ablatitium	Ab	8 0 0 5 9 6 3 8
Radix singularis		3
Homogenei residuum resoluendum		1 1 6 2 3 0 8 1 5 7 5
	— ggg	6 2 5 4
	— ff	1 0 2 4
	— 2.ffb	6 1 4 4 0
Radix singularis decupl. b = 30	4.bbb	1 0 8 0 0 0
	6.bb	5 4 0 0
	4.b	1 2 0
	+	1 1 3 5 2 0
	—	6 3 0 8 9 4
Diuisor	B	1 1 2 8 8 9 1 0 6
	— gggc	4 7 7 7 8
	— ffcc	5 0 1 7 6
	— 2ffbc	4 3 0 0 0 0
Rad. singul. secunda c = 7	4bbbc	7 5 6 0 0 0
	6.bbcc	2 6 4 6 0 0
	4.bccc	4 1 1 6 0
	cccc	2 4 0 1
	+	1 0 6 4 1 6 1
	—	4 8 5 6 3 3 8
Ablatitium	Bc	1 0 5 9 3 1 4 6 6 2
Radix aucta		3 · 7
Homogen. residuum resoluendum		1 0 2 9 9 3 4 9 5 5

Radix

Radix aucta		3 . 7
Homogenei reliquum resoluendum		1029934955
	$-ggg$	6254
	$-ff$	1024
	$-2.ffb$	257760
Radix aucta decup. $b = 370$	$4.bbb$	202612000
	$6.bb$	821400
	$4.b$	1480
	$+$	203441138
	$-$	758784
Diuisor	B	202319042
	$-gggc$	31270
	$-ffcc$	25600
	$-2.ffbc$	3788800
Rad. singularis tertia $c = 5$	$4.bbbc$	1013060000
	$6.bbcc$	20535000
	$4bccc$	185000
	$cccc$	625
	$+$	1033780625
	$-$	3845670
Ablatitium	Bc	1029934955
Radix uniuersalis complete educta		3 . 7 . 5
Homogenei residuum finale		00000000000

Ad æquationes quinti ordinis resoluendas.

PROBLEMA 15.

E dato æquationis simplicis sursolidi siue quinti ordinis . . . $aaaaa = lllll$. in numeris propositæ homogeneo radicem radicis quæsititiæ a. valorem analyticè educere.

Sit æquatio numerosè proposita . . . . $aaaaa = 15755509298176$.

Ergo . . . . $lllll = 15755509298176$.

Ponatur . . . . . $b+c = a$.

Ergo . . . . . $\left.\begin{array}{l} b+c \\ b+c \\ b+c \\ b+c \\ b+c \end{array}\right| = lllll.$

Factis

Factis igitur homogeneis particularibus,
fit . . . . . $+ .. bbbbb$ ═ $lllll$ ═ 15755509298176.
$+ 5.bbbbc$
$+ 10.bbbcc$
$+ 10.bbccc$
$+ 5.bcccc$
$+ ..ccccc$

Atque iisdem bifariàm ad hunc modum distributis,
fit . . . $\underbrace{+ bbbbb}_{Ab}$ $\underbrace{\begin{array}{l} + .5.bbbbc \\ + 10.bbbcc \\ + 10.bbccc \\ + .5.bcccc \\ + ..ccccc \end{array}}_{Bc}$ ═ 15755509298176.

Est autem æquationis huius pars speciosa bipartita $Ab. Bc.$ resolutionis canon, cuius applicatione operis analytici processus dirigendus est. Quem ritè constitutum esse per explicationis congruentiam superiorum Lemmatum exemplo demonstrari potest.

Factà igitur canonis huius applicatione omninò vt in subiecto schematismo ordinata conspicitur, fiat ipsius directione dati homogenei 15755509298176. ad radicem ex eo educendam exempli resolutio, vt sequitur.

Radix vniuersalis successiuè educenda					4					3					6
Homogeneum resoluendum		1	5	7	5	5	5	0	9	2	9	8	1	7	6
Diuisor	bbbb		2	5	6										
Rad. sing. prima b = 4.															
Ablatitium	bbbbb	1	0	2	4										
Radix singularis					4										
Homogen. reliquum resoluend.			5	5	1	5	5	0	9	2	9	8	1	7	6
	5.bbbb		1	2	8	0	0	0	0	0					
	10.bbb				6	4	0	0	0	0					
Rad sing dec. b = 40	10.bb					1	6	0	0	0					
	5.b							2	0	0					
Diuisor	B		1	3	4	5	.	.	.	.					
	5.bbbbc		3	8	4	0	0	0	0	0					
	10.bbbcc			5	7	6	0	0	0	0					
Rad. sing. secund. c = 3	10.bbccc				4	3	2	0	0	0					
	5.bcccc					1	6	2	0	0					
	..ccccc							2	4	3					
Ablatitium	Bc		4	4	6	0	8	4	4	3					
Radix aucta					4					3					
Homogenei reliquum resoluendum			1	0	5	4	6	6	4	9	9	8	1	7	6

Radix aucta		4 . . 3 . .
Homogenei residuum resoluendum		1054664998176
Rad. aucta dec. $b = 430$	$5.bbbb$	170940050000
	$10.bbb$	995070000
	$10.bb$	1849000
	$5.b$	1480
Diuisor	B	1717
	$5.bbbbc$	1025640300000
	$10.bbbcc$	28622520000
Rad. sing. tertia $c = 6$	$10.bbccc$	399384000
	$4\,bcccc$	2786400
	$ccccc$	7776
Ablatitium	Bc	1054664998176
Radix vniuersalis complete educta		4 . . 3 . . 6
Homogenei residuum finale		0000000000000

E dato igitur homogeneo 15755509298176. factâ ipsius ad hunc modum resolutione educta est radix 436. radici quæsititiæ a. æqualis, quæ educenda erat.

PROBLEMA 16.

E dato æquationis . . . $aaaaa - ffaaa + hhhha = lllll$. . . . in numeris propositæ homogeneo radicem radicis quæsititiæ a. valorem analyticè educere.

Sit æquatio numeris proposita . . . . . $aaaaa - 57.aaa + 5263.a = 9000505558322$.

Ergo . . . . $ff = 57$. & $hhhh = 5263$. & $lllll = 9000505558322$.

Ponatur . . . . . $b + c = a$.

Ergo . . . . . $+ \left| \begin{matrix} b+c \\ b+c \\ b+c \\ b+c \\ b+c \end{matrix} \right. - \left| \begin{matrix} ff \\ b+c \\ b+c \\ b+c \end{matrix} \right. + \left| \begin{matrix} hhhh \\ b+b \end{matrix} \right. = 9000505558322.$

Factis igitur homogeneis particularibus,

fit . . . . $+ ..bbbbb - .ffbbb + hhhhb = 9000505558322.$

$+ 5.bbbbc - 3.ffbbc + hhhhc$

$+ 10.bbbcc - 3.ffbcc$

$+ 10.bbccc - ..ffccc$

$+ 5.bcccc$

$+ ..ccccc$

Atque

Et ijsdem bifariam ad hunc modum distributis,

fit . . . . $+ hhhhb + hhhhc + .3.ffbbc + 10.bbccc = 90050558322$.
$- ffbbb - .ffccc + 5.bbbbc + 5.bcccc$
$+ bbbbb - 3.ffbcc + 10.bbbcc + .ccccc$

$$\underbrace{+hhhhb - ffbbb + bbbbb}_{Ab} \quad \underbrace{+hhhhc - .ffccc - 3.ffbcc + 3.ffbbc + 5.bbbbc + 10.bbbcc + 10.bbccc + 5.bcccc + .ccccc}_{Bc.}$$

Est autem æquationis huius pars speciosa bipartita *Ab. Bc.* resolutionis canon, cuius applicatione operis analytici processus dirigendus est. Quem ritè constitutum esse superiorum Lemmatum exemplo demonstrari potest.

Factâ igitur canonis huius applicatione, omninò vt in subiecto schematismo ordinata conspicitur, fiat ipsius directione dati homogenei 900050558322. ad radicem ex eo educendam resolutio, vt sequitur.

		2 4 6
Homogeneum resoluendum	*lllll* .	9 0 0 0 5 0 5 5 8 3 2 2
	hhhh	. . . . 0 5 2 6 3
	$-$*ff* . $-$.	0 0 0 5 7
	bbbb .	1 6
	$+$.	2 6 0 0 0 0 0 . . . .
Diuisor	*A* . .	1 5 9 9 4 . . . .
	hhhhb .	. . . . 1 0 5 2 6
$b = 2$	$-$*ffbbb* .	$-$ 0 0 4 5 6
	bbbbc .	3 2
	$+$.	3 2 0 0 0 1 0 5 2 6
Ablatitium	. *Bc* .	3 1 9 5 4 5 0 5 2 6
		2
Homogenei reliquum resoluendum		5 8 0 5 0 5 5 0 5 7 2 2
	hhhh	. . . . 5 2 6 3
	$-$. *ff*	. . . $-$ . . . . . 1 . . 5 7
	$-$3*ffb*	. . $-$ 3 4 2 0
$b = 20$	$-$3.*ffbb*	. $-$ 6 8 4 0 0
	5.*bbbb*	. . 8 0 0 0 0 0
	10.*bbb*	. . . . . 8 0 0 0 0
	10.*bb*	. . . . . . 4 0 0 0
	5.*b*	. . . . 1 0 0
	$+$ . .	8 8 4 1 0 0 . . . . . .
	$-$	7 1 8 . . . . .
Diuisor	*B* . . .	8 8 3 3 8 . . . .
	hhhhc	. . . . 1 1 0 5 2
	$-$*ffccc*	. $-$ 3 6 4 8
	$-$3.*ffbcc*	. $-$ 5 4 7 2 0
	$-$3.*ffbbc*	. $-$ . . 2 7 3 6 0 0
$c = 4$	5.*bbbbc* .	3 7 2 0 0 0 0
	10.*bbbcc* .	1 2 8 0 0 0 0
	10.*bbccc* .	. 2 5 6 0 0 0
	5.*bcccc* .	. . . 2 5 6 0 0
	ccccc .	. . . . 1 0 2 4
	$+$ . . .	4 7 6 2 6 2 6 1 0 5 2
	$-$ . . .	. . . 3 3 1 9 6 8
Ablatitium	*Bc* . .	4 7 5 9 3 0 6 4 2 5 2
		2 4
Homogeneum reliquum resoluendum		1 0 4 5 7 4 8 6 3 2 0 2

Homoge-

			2					4					
Homogenei reliquum resoluendum		1	0	4	5	7	4	8	6	3	2	0	2
	$hhhh$									5	2	6	3
	$-\,ff$											5	7
	$-\,3.ffb$								4	1	0	4	0
	$-\,3ffbb$						9	8	4	9	6	0	0
b = 240	$5.bbbb$		1	6	5	8	8	8	0	0	0	0	0
	$10.bbb$				1	3	8	2	4	0	0	0	0
	$10.bb$							9	7	6	0	0	0
	$5.b$									1	2	0	0
	$+$		1	6	7	2	7	7	2				
	$-$						9	8	9				
Diuisor	B		1	6	7	1	7						
	$hhhhc$								3	1	5	7	8
	$-\,ffccc$								1	2	3	1	2
	$-\,3ffbcc$						1	4	7	7	4	4	0
	$-\,3.ffbbc$					5	9	0	9	7	6	0	0
b = 6	$5.bbbbc$		9	9	5	3	2	8	0	0	0	0	0
	$10.bbbcc$			4	9	7	6	6	4	0	0	0	0
	$10.bbccc$				1	2	4	4	1	6	0	0	0
	$5.bcccc$						1	5	5	5	2	0	0
	$ccccc$									7	7	7	6
	$+$	1	0	4	6	3	5	4	5	0	5	5	4
	$-$					6	0	5	8	7	3	5	2
Ablatitium	Bc	1	0	4	5	7	4	8	6	3	2	0	2
Radix vniuersalis complete educta			2					4					6
Homogenei reliquum finale			0	0	0	0	0	0	0	0	0	0	0

E dato igitur homogeneo 9000505558322. factâ ipsius ad hunc modum resolutione educta est radix 246. radici quæsititiæ a. æqualis, quæ educenda erat.

Approximationis artificium in casu asymmetriæ.

Quoniam accidit communitèr radicem è dato homogeneo educendam irrationalem siue numero inexplicabilem esse, nullo scilicet analytices artificio educibilem, quin ad finem operis supersit semper residuum aliquod imperfectæ resolutionis indicium. In eo casu qui frequentissimus est, residuum finale per multiplicationem gradualem additis ad dextram cyphris producendum est, binarijs in genere quadratico, ternarijs in cubico, quaternarijs in quadratico &c. quotcunquè libuerit, & radix iam educta inuariato operis tenore ad decimas, centesimas, millesimas, &c. vnitatis partes, ad requisitam scilicet veritati proximitatem continuanda est. Approximationis huius processum quùm simplex & vniformis sit, exemplis sequentibus ostendisse sufficiat.

Approximationis exemplum quadraticum.

Æquatio resoluenda . . . $\begin{cases} aa + da = ff \\ aa + 14.a = 7929. \end{cases}$

Canon

Resolutionis canon . . . . $\left| \begin{array}{ll} +db & +.dc \\ +bb & +2.bc \\ \underbrace{}_{Ab} & +.cc \\ & \underbrace{}_{Bc.} \end{array} \right.$

Homogeneum resoluendum . . . 8 2
7 9 2 9

d 1 4
b 8
Diuisor . . . . A 9 4
db . . . 1 1 2
b ═══ 8 . . . bb . . . 6 4
Ablatitium . . . . Ab . . . 7 5 2

Homogenei residuum . . . 8
4 0 9

d 1 4
b ═══ 80 . . . 2b 1 6 0
Diuisor . . . . B 1 7 4
dc 2 8
c ═══ 2 . . . 2.bc 3 2 0
cc 4
Ablatitium . . . . Bc 3 5 2

Rad. ad vnitatem educta . . . 8 2
Homogen. residuum finale productum . . . 5 7 0 0 0 0 0 0

d 1 4
b ═══ 820 . . . 2.b 1 6 4 0
Diuisor . . . . B 1 7 8 0
dc 4 2
c ═══ 3 . . . 2.bc 4 9 2 0
cc 9
Ablatitium . . . . Bc 5 3 4 9

8 2 3
Homogen. producti residuum . . . 3 5 1 0 0 0 0

d 1 4
b ═══ 8230 . . . 2.b 1 6 4 6 0
Diuisor . . . . A. 1 7 8 6 0
dc 1 4
c ═══ 1 . . . 2.bc 1 6 4 6 0
cc 1
Ablatitium . . . . Bc 1 7 8 6 6

8 2 3 1
Homogen. producti residuum . . . 1 7 2 3 9 0 0

			8	2		3		1			
Homogen. producti residuum				1	7	2	3	9	0	0	

		d				1	4				
b — 82310		$2b$				1	6	4	6	2	0
Diuisor		B	1			1	7	8	6	2	0
		dc				1	2	6			
c — 9		$2.bc$			1	4	8	1	5	8	0
		cc								8	1
Ablatitium		Bc			1	6	0	7	6	6	1

Radix ad 1000^mas. continuata	8	2		3		1		9
Homog producti residuum abiectaneum			1	1	6	2	3	9

Factâ ad hunc modum dati homogenei *7929.* analysi habetur æquationis propositæ radix $\frac{82319}{1000}$ siue $82\frac{319}{1000}$ scilicet intra vnitatis partem milesimam verę proxima.

Approximationis exemplum cubicum.

Æquatio resoluenda . . . $\begin{cases} aaa + ffa = ggg \\ aaa + 135.a = 98754. \end{cases}$

Resolutionis canon . . . . $\left| \begin{array}{l} + ffb + .ffc + 3.bcc \\ \underbrace{+ bbb}_{Ab} \underbrace{+ 3bbc + .ccc}_{Bc} \end{array} \right.$

Radix				4			5
Homogeneum resoluendum	ggg		9	8	7	5	4

	ff			1	3	5
	bb		1	6		
Diuisor	A		1	7	3	5
	ffb		5	4	0	
b — 4	bbb	6	4			
Ablatitium	Ab	6	9	4	0	

		4			
Homog. residuum resoluendum	2	9	3	5	4

Homog.

Homogenei residuum resoluendum		2 9 3 5 4
	ff	1 3 5
b = 40	3. *bb*	4 8 0 0
	3. *b*	1 2 0
Diuisor	*B*	5 0 5
	ffc	6 7 5
	3. *bbc*	2 4 0 0 0
b = 5	3. *bcc*	3 0 0 0
	ccc	2 2 5
Ablatitium	*Bc*	2 7 8 0 0

Radix ad vnitatem educta		4 5
Homogenei residuum finale productum		1 5 5 4 0 0 0 0 0 0
	ff	1 3 5
b = 450	3. *bb*	6 0 7 5 0 0
	3. *b*	1 3 5 0
Diuisor	*B*	6 2 2
	ffc	2 7 0
b = 2	3. *bbc*	1 2 1 5 0 0 0
	3. *bcc*	5 4 0 0
	ccc	8
Ablatitium	*B*	1 2 4 7 4 0 8

		4 5 2
Homogen. producti residuum		3 0 6 5 9 2 0 0 0
	ff	1 3 5
c = 4520	3. *bb*	6 1 2 9 1 2 0 0
	3. *b*	1 3 5 6 0
Diuisor	*B*	6 2 6 5 4
	ffc	5 4 0
	3. *bbc*	2 4 5 1 6 4 8 0 0
c = 4	3. *bcc*	2 1 6 9 6 0
	ccc	6 4
Ablatitium	*Bc*	2 5 0 7 8 1 8 2 4

Radix ad 100mas. continuata	4 5 2 4
Homogenei producti residuum abiectaneum	5 5 8 1 0 1 7 6

Factâ ad hunc modum dati homogenei 98754. analisi habetur æquationis propositæ radix $\frac{4524}{100}$ siue 45 $\frac{24}{100}$ hoc est, intra vnitatis partem centesimam veræ proxima.

CANO-

CANONES DIRECTORII.

Æqquationum Quadraticarum, Cubicarum, & Biquadraticarum, cum canonibus suis operis analytici directorijs cuique adscriptis integra & ordinata synopsis. In qua species canonicæ, vt operi analytico distinctè respondeant, quadripartitò descriptæ sunt, antecedentium schematismorum exemplo, videlicet duæ primæ partes *A.* & *Ab.* pro primæ radicis singularis eductione *A.* pro diuisore *Ab.* pro ablatitio; & duæ secundæ partes *B.* & *Bc*, pro secundarijs reliquis, *B.* pro diuisore, *Bc.* pro ablatitio.

Quadraticarum casus siue differentiæ. 4.

Prima.

$$aa = ff.$$

$$A \ldots b \qquad B \ldots 2.b$$

$$Ab \ldots bb \qquad Bc \ldots \left\{\begin{array}{l} 2.bc \\ .cc \end{array}\right.$$

2.

$$aa + da = ff.$$

$$A \left\{\begin{array}{l} d \\ b \end{array}\right. \qquad B \left\{\begin{array}{l} .d \\ 2.b \end{array}\right.$$

$$Ab \left\{\begin{array}{l} db \\ bb \end{array}\right. \qquad Bc \left\{\begin{array}{l} .dc \\ 2.bc \\ ..cc \end{array}\right.$$

3.

$$aa - da = ff.$$

$$A.. \left\{\begin{array}{l} -d \\ ..b \end{array}\right. \qquad B. \left\{\begin{array}{l} -d \\ 2.b \end{array}\right.$$

$$Ab \left\{\begin{array}{l} -db \\ bb \end{array}\right. \qquad Bc \left\{\begin{array}{l} -dc \\ 2.bc \\ ..cc \end{array}\right.$$

4.

$$-aa + da = ff.$$

$$A. \left\{\begin{array}{l} ..d \\ -b \end{array}\right. \qquad B. \left\{\begin{array}{l} ...d \\ -2.b \end{array}\right.$$

$$Ab \left\{\begin{array}{l} ..db \\ -bb \end{array}\right. \qquad Bc \left\{\begin{array}{l} ...dc \\ -2.bc \\ -.cc \end{array}\right.$$

Cubicarum differentiæ. 14.

Prima.

$$aaa = ggg.$$

$$A \ldots bb \qquad B. \left\{\begin{array}{l} 3.bb \\ 3.b \end{array}\right.$$

$$Ab \ldots bbb \qquad Bc \left\{\begin{array}{l} 3.bbc \\ 3.bcc \\ .ccc \end{array}\right.$$

2.

$$aaa + ffa = ggg.$$

$$A.. \left\{\begin{array}{l} ff \\ bb \end{array}\right. \qquad B.. \left\{\begin{array}{l} .ff \\ 3.bb \\ 3.b \end{array}\right.$$

$$Ab \left\{\begin{array}{l} ffb \\ bbb \end{array}\right. \qquad Bc \left\{\begin{array}{l} ..ffc \\ 3.bbc \\ 3.bcc \\ .ccc \end{array}\right.$$

3.

$$aaa - ffa = ggg.$$

$$A. \left\{\begin{array}{l} -ff \\ .bb \end{array}\right. \qquad B \left\{\begin{array}{l} -ff \\ 3.bb \\ 3.b \end{array}\right.$$

$$Ab \left\{\begin{array}{l} -ffb \\ ..bbb \end{array}\right. \qquad Bc \left\{\begin{array}{l} -ffc \\ 3\ bbc \\ 3.bcc \\ ..ccc \end{array}\right.$$

$-aaa$

4.

$$-aaa + ffa = ggg.$$

$$A..\left\{\begin{array}{l} ..ff \\ -bb \end{array}\right. \qquad Ab\left\{\begin{array}{l} ..ffb \\ -bbb \end{array}\right.$$

$$B.\left\{\begin{array}{l} ....ff \\ -3.bb \\ -3.b \end{array}\right. \qquad Bc\left\{\begin{array}{l} ..ffc \\ -3.bbc \\ -3.bcc \\ -.ccc \end{array}\right.$$

5.

$$aaa + daa = ggg.$$

$$A.\left\{\begin{array}{l} .d \\ bb \end{array}\right. \qquad Ab\left\{\begin{array}{l} dbb \\ bbb \end{array}\right.$$

$$B\left\{\begin{array}{l} .d \\ 2.db \\ 3.bb \\ 3.b \end{array}\right. \qquad Bc\left\{\begin{array}{l} .dcc \\ 2.dbc \\ 3.bbc \\ 3.bcc \\ ..ccc \end{array}\right.$$

6.

$$aaa - daa = ggg.$$

$$A.\left\{\begin{array}{l} -d \\ .bb \end{array}\right. \qquad Ab\left\{\begin{array}{l} -dbb \\ .bbb \end{array}\right.$$

$$B\left\{\begin{array}{l} -..d \\ -2.db \\ .3bb \\ .3b \end{array}\right. \qquad Bc\left\{\begin{array}{l} -dcc \\ 2.dbc \\ 3.bbc \\ 3.bcc \\ ccc \end{array}\right.$$

7.

$$-aaa + daa = ggg.$$

$$A\left\{\begin{array}{l} d \\ -bb \end{array}\right. \qquad Ab\left\{\begin{array}{l} dbb \\ -bbb \end{array}\right.$$

$$B.\left\{\begin{array}{l} d \\ 2.db \\ -3.bb \\ -3b \end{array}\right. \qquad Bc\left\{\begin{array}{l} ..dcc \\ 2.dbc \\ -3.bbc \\ -3.bcc \\ -..ccc \end{array}\right.$$

8.

$$aaa + daa + ffa = ggg.$$

$$A\left\{\begin{array}{l} ff \\ d \\ db \end{array}\right. \qquad Ab\left\{\begin{array}{l} ffb \\ dbb \\ bbb \end{array}\right.$$

$$B\left\{\begin{array}{l} .ff \\ .d \\ 2\ db \\ 3.bb \\ 3.b \end{array}\right. \qquad Bc\left\{\begin{array}{l} .ffc \\ .dcc \\ 2.dbc \\ 3.bbc \\ 3.bcc \\ ccc \end{array}\right.$$

9.

$$aaa + daa - ffa = ggg.$$

$$A\left\{\begin{array}{l} -ff \\ d \\ bb \end{array}\right. \qquad Ab\left\{\begin{array}{l} ffb \\ dbb \\ bbb \end{array}\right.$$

$$B\left\{\begin{array}{l} -ff \\ d \\ 2.db \\ 3.bb \\ 3.b \end{array}\right. \qquad Bc\left\{\begin{array}{l} -ff \\ dcc \\ 2.dbc \\ 3\ bbc \\ 3.bcc \\ ccc \end{array}\right.$$

10.

$$-aaa - daa + ffa = ggg.$$

$$A\left\{\begin{array}{l} ff \\ -d \\ -bb \end{array}\right. \qquad Ab\left\{\begin{array}{l} ffb \\ -dbb \\ -bbb \end{array}\right.$$

$$B\left\{\begin{array}{l} ff \\ -d \\ -2.db \\ -3.bb \\ -3.b \end{array}\right. \qquad Bc\left\{\begin{array}{l} .ffc \\ -dcc \\ -2\ dbc \\ -3\ bbc \\ -3\ bcc \\ -\ ccc \end{array}\right.$$

CANONES DIRECTORII.

ÆQquationum Quadraticarum, Cubicarum, & Biquadraticarum, cum canonibus suis operis analytici directorijs cuique adscriptis integra & ordinata synopsis. In qua species canonicæ, vt operi analytico distinctè respondeant, quadripartitò descriptæ sunt, antecedentium schematismorum exemplo, videlicet duæ primæ partes *A.* & *Ab.* pro primæ radicis singularis eductione *A.* pro diuisore *Ab.* pro ablatitio; & duæ secundæ partes *B.* & *Bc.* pro secundarijs reliquis, *B.* pro diuisore, *Bc.* pro ablatitio.

Quadraticarum casus siue differentiæ. 4.

Prima.

$$aa = ff.$$

$A \;...\; b$ $\qquad$ $B \;...\; 2.b$

$Ab \;..\; bb$ $\qquad$ $Bc \;..\; \left\{\begin{matrix} 2.bc \\ .cc \end{matrix}\right.$

2.

$$aa + da = ff.$$

$A \left\{\begin{matrix} d \\ b \end{matrix}\right.$ $\qquad$ $B \left\{\begin{matrix} .d \\ 2.b \end{matrix}\right.$

$Ab \left\{\begin{matrix} db \\ bb \end{matrix}\right.$ $\qquad$ $Bc \left\{\begin{matrix} .dc \\ 2.bc \\ ..cc \end{matrix}\right.$

3.

$$aa - da = ff.$$

$A.. \left\{\begin{matrix} -d \\ ..b \end{matrix}\right.$ $\qquad$ $B. \left\{\begin{matrix} -d \\ 2.b \end{matrix}\right.$

$Ab \left\{\begin{matrix} -db \\ bb \end{matrix}\right.$ $\qquad$ $Bc \left\{\begin{matrix} -dc \\ 2.bc \\ ..cc \end{matrix}\right.$

4.

$$-aa + da = ff.$$

$A. \left\{\begin{matrix} ..d \\ -b \end{matrix}\right.$ $\qquad$ $B. \left\{\begin{matrix} ...d \\ -2.b \end{matrix}\right.$

$Ab \left\{\begin{matrix} ..db \\ -bb \end{matrix}\right.$ $\qquad$ $Bc \left\{\begin{matrix} ...dc \\ -2.bc \\ -.cc \end{matrix}\right.$

Cubicarum differentiæ. 14.

Prima.

$$aaa = ggg.$$

$A \;....\; bb$ $\qquad$ $B. \left\{\begin{matrix} 3.bb \\ 3.b \end{matrix}\right.$

$Ab. \;..\; bbb$ $\qquad$ $Bc \left\{\begin{matrix} 3.bbc \\ 3.bcc \\ .ccc \end{matrix}\right.$

2.

$$aaa + ffa = ggg.$$

$A.. \left\{\begin{matrix} ff \\ bb \end{matrix}\right.$ $\qquad$ $B.. \left\{\begin{matrix} .ff \\ 3.bb \\ 3.b \end{matrix}\right.$

$Ab \left\{\begin{matrix} ffb \\ bbb \end{matrix}\right.$ $\qquad$ $Bc \left\{\begin{matrix} ..ffc \\ 3.bbc \\ 3.bcc \\ .ccc \end{matrix}\right.$

3.

$$aaa - ffa = ggg.$$

$A. \left\{\begin{matrix} -ff \\ .bb \end{matrix}\right.$ $\qquad$ $B \left\{\begin{matrix} -ff \\ 3.bb \\ 3.b \end{matrix}\right.$

$Ab \left\{\begin{matrix} -ffb \\ ..bbb \end{matrix}\right.$ $\qquad$ $Bc \left\{\begin{matrix} -ffc \\ 3.bbc \\ 3.bcc \\ ..ccc \end{matrix}\right.$

4.

$$-aaa+ffa = ggg.$$

$$A..\left\{\begin{array}{l} ..ff \\ -bb \end{array}\right. \qquad B.\left\{\begin{array}{l} ....ff \\ -3.bb \\ -3.b \end{array}\right.$$

$$Ab\left\{\begin{array}{l} ..ffb \\ -bbb \end{array}\right. \qquad Bc\left\{\begin{array}{l} ..ffc \\ -3.bbc \\ -3.bcc \\ -.ccc \end{array}\right.$$

5.

$$aaa+daa = ggg.$$

$$A.\left\{\begin{array}{l} .d \\ bb \end{array}\right. \qquad B\left\{\begin{array}{l} .d \\ 2.db \\ 3.bb \\ 3.b \end{array}\right.$$

$$Ab\left\{\begin{array}{l} dbb \\ bbb \end{array}\right. \qquad Bc\left\{\begin{array}{l} .dcc \\ 2.dbc \\ 3.bbc \\ 3.bcc \\ ..ccc \end{array}\right.$$

6.

$$aaa-daa = ggg.$$

$$A.\left\{\begin{array}{l} -d \\ .bb \end{array}\right. \qquad B\left\{\begin{array}{l} -..d \\ -2.db \\ .3bb \\ .3b \end{array}\right.$$

$$Ab\left\{\begin{array}{l} -dbb \\ .bbb \end{array}\right. \qquad Bc\left\{\begin{array}{l} -dcc \\ 2.dbc \\ 3.bbc \\ 3.bcc \\ ccc \end{array}\right.$$

7.

$$-aaa+daa = ggg.$$

$$A\left\{\begin{array}{l} d \\ -bb \end{array}\right. \qquad B.\left\{\begin{array}{l} d \\ 2.db \\ -3.bb \\ -3b \end{array}\right.$$

$$Ab\left\{\begin{array}{l} dbb \\ -bbb \end{array}\right. \qquad B:\left\{\begin{array}{l} ..dcc \\ 2.dbc \\ -3.bbc \\ -3.bcc \\ -..ccc \end{array}\right.$$

8.

$$aaa+daa+ffa = ggg.$$

$$A\left\{\begin{array}{l} ff \\ d \\ db \end{array}\right. \qquad B\left\{\begin{array}{l} .ff \\ .d \\ 2\,db \\ 3.bb \\ 3.b \end{array}\right.$$

$$Ab\left\{\begin{array}{l} ffb \\ dbb \\ bbb \end{array}\right. \qquad Bc\left\{\begin{array}{l} .ffc \\ .dcc \\ 2.dbc \\ 3.bbc \\ 3.bcc \\ ccc \end{array}\right.$$

9.

$$aaa+daa-ffa = ggg.$$

$$A\left\{\begin{array}{l} -ff \\ d \\ bb \end{array}\right. \qquad B\left\{\begin{array}{l} -ff \\ d \\ 2.db \\ 3.bb \\ 3.b \end{array}\right.$$

$$Ab\left\{\begin{array}{l} ffb \\ dbb \\ bbb \end{array}\right. \qquad Bc\left\{\begin{array}{l} -ff \\ dcc \\ 2.dbc \\ 3\,bbc \\ 3.bcc \\ ccc \end{array}\right.$$

10.

$$-aaa-daa+ffa = ggg.$$

$$A\left\{\begin{array}{l} ff \\ -d \\ -bb \end{array}\right. \qquad B\left\{\begin{array}{l} ff \\ -d \\ -2.db \\ -3.bb \\ -3.b \end{array}\right.$$

$$Ab\left\{\begin{array}{l} ffb \\ -dbb \\ -bbb \end{array}\right. \qquad Bc\left\{\begin{array}{l} .ffc \\ -dcc \\ -2\,dbc \\ -3\,bbc \\ -3\,bcc \\ -ccc \end{array}\right.$$

11.

aaa — daa + ffa ═══ *ggg.*

A { *ff* / *— d* / *bb*

Ab { *ffb* / *— dbb* / *bbb*

B. { *. ff* / *— d* / *— 2.db* / *3. bb* / *3 b*

Bc { *. . ffc* / *— . dcc* / *— 2.dbc* / *3.bbc* / *3.bcc* / *. . ccc*

12.

— aaa + daa — ffa ═══ *ggg.*

A { *— ff* / *. d* / *— bb*

Ab { *— ff* / *. dbb* / *— bbb*

B. { *— . . ff* / *. . d* / *2. db* / *— 3.bb* / *— 3. b*

Bc { *— . . ffc* / *. . dcc* / *. 2 dbc* / *— 3.bbc* / *— 3.bcc* / *— . ccc*

13.

aaa — daa — ffa ═══ *ggg.*

A { *— ff* / *— d* / *bb*

Ab { *— ffb* / *— dbb* / *bbb*

B. { *— . ff* / *— . d* / *— 2.db* / *.3.bb* / *.3.b*

Bc { *— . ffc* / *— . dcc* / *— 2.dbc* / *3.bbc* / *3.bcc* / *. . ccc*

14.

— aaa + daa + ffa ═══ *ggg.*

A { *. ff* / *. . d* / *— bb*

A b { *. ffb* / *. dbb* / *— bbb*

B { *. . ff* / *. . d* / *. 2.db* / *— 3.bb* / *— 3.b*

Bc { *. . ffc* / *. . dcc* / *. 2.dbc* / *— 3.bbc* / *— 3.bcc* / *— . ccc*

Biquadraticarum differentiæ. 46.

Prima.

aaaa ═══ *bbbb*

A . . . *bbb*

Ab . . *bbbb*

B { *4.bbb* / *6.bb* / *4.b*

Bc { *4.bbbc* / *6.bbcc* / *4.bccc* / *. cccc*

2.

aaaa + ggga ═══ *bbbb.*

A { *ggg* / *bbb*

Ab { *gggb* / *bbbb*

B { *. ggg* / *4. bbb* / *6.bb* / *4 b*

Bc { *. gggc* / *4.bbbc* / *6.bbcc* / *4 bccc* / *. cccc*

3.

aaaa — ggga ═══ hhhh.

A { — ggg
. bbb

Ab { — gggb
. bbbb

B { — . ggg
. . 4 bbb
6. bb
4 b

Bc { — . gggc
. 4 bbbc
. 6 bbcc
. 4 bccc
. . cccc

4.

— aaaa + ggga ═══ hhhh.

A { . ggg
— bbb

Ab { . gggb
— bbbb

B { . . ggg
— 4 bbb
— 6 bb
— 4. b

Bc { . . gggc
— 4. bbbc
— 6. bbcc
— 4. bccc
— . . cccc

5.

aaaa + ffaa ═══ hhhh.

A { ff
bbb

Ab { . gggb
bbbb

B . { . . ff
2 ffb
4 bbb
6 bb
4. b

Bc { . . ffcc
2. ffbc
4. bbbc
6. bbcc
4. bccc
. cccc

6.

aaaa — ffaa ═══ hhhh.

A { — ff
. bbb

Ab { — ffbb
. bbbb

B . . { — . . ff
— 2. ffb
. 4 bbb
. 6. bb
. 4. b

Bc { — . ffcc
— 2. ffbc
. 4 bbbc
. 6. bbcc
. 4. bccc
. . cccc

7.

— aaaa + ffaa ═══ hhhh.

A { . ff
— bbb

Ab { . ff
— bbbb

B . { . ff
. 2. ffb
— 4. bbb
— 6. bb
— 4. b

Bc { . . ffcc
. 2. ffbc
— 4 bbbc
— 6. bbcc
— 4 bccc
— . cccc

8.

aaaa + daaa ═══ hhhh.

A { d
bbb

Ab { dbbb
bbbb

B { . d
3. db
3 dbb
4 bbb
6 bb
4. b

Bc { . dccc
3. dbcc
3. dbbc
4. bbbc
6. bbcc
4. bccc
. . cccc

9

aaaa — daaa == bbbb.

A { — d
bbb }

Ab { — dbbb
. bbbb }

B { — . . d
— 3. db
— 3. dbb
. 4 bbb
. 6 bb
. 4 b }

Bc { — . . dccc
— 3. dbcc
— 3 dbbc
. 4 bbbc
. 6 bbcc
. 4 bccc
. . cccc }

10.

— aaaa + daaa == bbbb.

A { . d
— bbb }

Ab { . dbbb
— bbbb }

B. { . . d
. 3 db
. 3 dbb
— 4 bbb
— 6. bb
— 4 b }

Bc { . . dccc
. 3 dbcc
. 3 dbbc
— 4. bbbc
— 6 bbcc
— 4 bccc
— . . cccc }

11.

aaaa + ffaa + ggga == bbbb.

A { ggg
ff
bbb }

Ab { gggb
ffbb
bbbb }

B { . ggg
. ff
2. ffb
4 bbb
6. bb
4. b }

Bc { . gggc
. ffcc
2. ffbc
4 bbbc
6. bbcc
4. bccc
. cccc }

12.

aaaa + ffaa — ggga == bbbb.

A { — ggg
. ff
. bbb }

Ab { — gggb
. ffbb
. bbbb }

B { — ggg
. ff
2. ffb
4. bbb
6. bb
4. b }

Bc { — gggc
. ffcc
. 2 ffbc
4. bbbc
6 bbcc
4 bccc
. cccc }

13.

— aaaa — ffaa + ggga == bbbb.

A { . ggg
— ff
— bbb }

Ab { . gggb
— ffbb
— bbbb }

B { . . ggg
— . ff
— 2. ffb
— 4 bbb
— 6 bb
— 4. b }

Bc { . gggc
— . ffcc
— 2. ffbc
— 4 bbbc
— 6. bbcc
— 4 bccc
— . cccc }

14.

aaaa — ffaa + ggga == bbbb.

A { . ggg
— ff
. bbb }

Ab { . gggb
— ffbb
. bbbb }

B { . . ggg
— . ff
— 2. ffb
. 4 bbb
. 6 bb
. 4. b }

B { . gggc
— . ffcc
— 2. ffbc
. . 4 bbbc
. . 6. bbcc
. 4 bccc
. . cccc }

15.

— aaaa + ffaa — ggga ═══ bbbb.

A { — ggg
. ff
— bbb }

Ab { — gggb
. ffbb
— bbbb }

B { — . ggg
. ff
2. ffb
— 4. bbb
— 6. bb
— 4 b }

Bc { — . gggc
. . ffc
. . 2. ffbc
— 4. bbbc
— 6 bbcc
— 4. bccc
— cccc }

16.

aaaa — ffaa — ggga ═══ bbbb.

A { — ggg
— ff
. bbb }

Ab { — gggb
— ffbb
. bbbb }

B { — . . ggg
— . ff
— 2. ffb
. 4 bbb
. 6. bb
. 4. b }

Bc { — gggc
— . ffcc
— 2. ffbc
. . 4 bbbc
. . 6. bbcc
. 4. bccc
. . cccc }

17.

— aaaa + ffaa + ggga ═══ bbbb.

A { . ggg
. ff
— bbb }

Ab { . gggb
. ffbb
— bbbb }

B { . . ggg
. . ff
. 2. ffb
— 4. bbb
— 6. bb
— 4 b }

Bc { . gggc
. . ffcc
. 2. ffbc
— 4. bbbc
— 6. bbcc
— 4 bccc
— . cccc }

18.

aaaa + daaa + ggga ═══ bbbb.

A { . ggg
. d
. bbb }

Ab { . gggb
. dbbb
. bbbb }

B { . . ggg
. d
. 3. db
. 3. dbb
4. bbb
6. bb
4 b }

Bc { . gggc
. dccc
3 dbcc
3 dbbc
4 bbbc
6 bbcc
4 bccc
. cccc }

19.

aaaa + daaa — ggga ═══ bbbb.

A { — ggg
. . d
. . bbb }

Ab { — gggb
. dbbb
. . bbbb }

B { — . ggg
. . d
. 3 db
. 3 dbb
. 4 bbb
. 6 bb
. 4. b }

Bc { — . gggc
. 3 dbcc
. 3 dbbc
. 4 bbbc
. 6. bbcc
. 4 bccc
. cccc }

20.

— aaaa — daaa + ggga ═══ bbbb.

A { . . ggg
— d
— bbb }

Ab { . . gggb
— dbbb
— bbbb }

B { . . ggg
— . d
— 3. db
— 3. dbb
— 4. bbb
— 6. bb
— 4. b }

Bc { . . gggc
— . dccc
— 3. dbcc
— 3. dbbc
— 4. bbbc
— 6. bbcc
— 4. bccc
— . . cccc }

21.

aaaa — daaa + ggga = bbbb.

A { · ggg
— d
. bbb

Ab { · gggb
— dbbb
. bbbb

B. { · · ggg
— . d
— 3. db
— 3. dbb
. 4 bbb
. 6. bb
. 4. b

Bc { · · ggggc
— . . d cc
— 3. dbcc
— 3. dbbc
. 4 bbbc
. 6. bbcc
. 4 bccc
. . cccc

22.

— aaaa + daaa — ggga = bbbb.

A { — ggg
d
— bbb

Ab { — gggb
. dbbb
— bbbb

B { — . ggg
. . d
. 3. db
3. dbb
— 4. bbb
— 6. bb
— 4. b

Bc { — . gggc
. . dccc
. 3. dbcc
. 3. dbbc
— 4. bbbc
— 6. bbcc
— 4. bccc
— . cccc

23.

aaaa — daaa — ggga = bbbb.

A { — ggg
— d
— bbb

Ab { — gggb
— dbbb
. bbbb

B { — . . ggg
— . d
— 3. db
— 3. dbb
. 4 bbb
. 6 bb
: 4. b

Bc { — . gggc
— . dccc
— 3. dbcc
— 3. dbbc
. 4 bbbc
. . 6. bbcc
. 4. b cc
. . cccc

24.

— aaaa + daaa + ggga = bbbb.

A { · ggg
. d
— bbb

Ab { · gggb
. dbbb
— bbbb

B { . . ggg
. . d
. 3. db
. 3. dbb
— 4. bbb
— 6. bb
— 4. b

Bc { . . gggc
. . dccc
. 3. dbcc
. 3. dbbc
— 4. bbbc
— 6. bbcc
— 4. bccc
— . . cccc

25.

aaaa + daaa + ggga ═══ bbbb.

A { . ff
. d
. bbb

Ab { . ffbb
. dbbb
. bbbb

B { . ff
2.ffb
. d
3.db
3.dbb
4 bbb
6.bb
4. b

Bc { .ffcc
2 ffbc
. dccc
3.dbcc
3 dbbc
4.bbbc
6 bbcc
4 bccc
. cccc

26.

aaaa + daaa — ffaa ═══ bbbb.

A { — ff
. d
. bbb

Ab { — ffbb
. dbbb
. bbbb

B { — . ff
— 2 ffb
. . d
. 3.db
. 3 dbb
. 4.bbb
. 6.bb
. 4.b

Bc { — . ffcc
— 2 ffbc
. . dccc
. 3.dbcc
. 3 dbbc
. 4 bbbc
. 6 bbcc
. 4 bccc
. . cccc

27.

— aaaa — daaa + ffaa ═══ bbbb.

A { . ff
— d
— bbb

Ab { . ffbb
— d bb
— bbbb

B. { . . . ff
. 2.ffb
— . d
— 3. db
— 3. dbb
— 4 bbb
— 6. bb
— 4. b

Bc { . . . ffcc
. 2.ffbc
— . . dccc
— 3.dbcc
— 3 dbbc
— 4.bbbc
— 6.bbcc
— 4.bccc
— . cccc

28.

aaaa — daaa + ffaa ═══ bbbb.

A { . ff
— d
. bbb

Ab { . ffbb
— dbbb
. bbbb

B. { . . ff
. 2 ffb
— . . d
— 3 db
— 3. dbb
. 4.bbb
. 6.bb
. 4. b

Bc { . . ffcc
. 2.ffbc
— . . dccc
— 3.dbcc
— 3.dbbc
. 4 bbbc
. 6.bbcc
. 4.bccc
. . cccc

29.

— aaaa + daaa — ffaa == bbbb.

A { — ff
. d
— bbb }

Ab { — ffbb
. dbbb
— bbbb }

B { — .ff
— 2ffb
. . d
. 3.db
. 3.dbb
— 4.bbb
— 6.bb
— 4 b }

Bc { — .ffcc
— 2.ffbc
. . . dccc
. 3.dbcc
. 3.dbbc
— 4.bbbc
— 6.bbcc
— 4.bccc
. . cccc }

31.

aaaa — daaa — ffaa == bbbb.

A { — ff
— d
.bbb }

Ab { — ffbb
— dbbb
. bbbb }

B { — .ff
— 2ffb
— .d
— 3.db
— 3.dbb
. .4.bbb
6.bb
4.b }

Bc { — .ffcc
— 2ffb
— .dccc
— 3.dbcc
— 3.dbbc
. 4.bbbc
6.bbcc
. 4 bccc
. . cccc }

30.

— aaaa + daaa + ffaa == bbbb.

A { . ff
. d
— bbb }

Ab { . ffbb
. dbbb
— bbbb }

B { . . ff
. 2.ffb
. . d
. 3.db
. 3.dbb
— 4.bbb
— 6.bb
— 4.b }

Bc { . .ffcc
. 2ffbc
. .dccc
3.dbcc
3.dbbc
— 4.bbbc
— 6.bbcc
— 4.bccc
— . cccc }

32.

aaaa + daaa + ffaa + ggga == bbbb.

A { ggg
ff
d
bbb }

Ab { gggb
ffbb
dbbb
bbbb }

B { . .ggg
. .ff
2.ffb
. .d
3.db
3.dbb
4.bbb
6.bb
4 b }

Bc { .gggc
.ffcc
2.ffbc
. dccc
2.dbcc
3.dbbc
4.bbbc
6.bbcc
4.bccc
. cccc }

33.

aaaa + daaa + ffaa — ggga == bbbb.

A:
- — ggg
- . ff
- . d
- . bbb

Ab:
- — gggb
- . ffbb
- . dbbb
- . bbbb

B:
- — . ggg
- . . ff
- 2. ffb
- . d
- . 3. db
- . 3. dbb
- . . 4. bbb
- . . 6. bb
- . 4 b

Bc:
- — gggc
- . . ffcc
- . 2. ffbc
- . . dccc
- . 3. dbcc
- . 3. dbbc
- . 4. bbbc
- . 6. bbcc
- . 4. bccc
- . cccc

34.

— aaaa — daaa — ffaa + ggga == bbbb.

A:
- . ggg
- — ff
- — d
- — bbb

Ab:
- . gggb
- — ffbb
- — dbbb
- — bbbb

B:
- . . ggg
- — . . ff
- — 2. ffb
- — . d
- — 3. db
- — 3. dbb
- — 4. bbb
- — 6. bb
- — 4. b

Bc:
- . . . gggc
- — . . ffcc
- — 2. ffbc
- — . . dccc
- — 3. dbcc
- — 3. dbbc
- — 4. bbbc
- — 6. bbcc
- — 4. bccc
- — . cccc

35.

aaaa + daaa — ffaa + ggga == bbbb.

A:
- . ggg
- — ff
- . d
- . bbb

Ab:
- . gggb
- — ffbb
- . d bb
- . bbbb

B.:
- . . . ggg
- — . ff
- — 2. ffb
- . d
- 3. db
- 3. dbb
- 4. bbb
- 6. bb
- 4. b

Bc:
- . . . gggc
- — ffcc
- — 2. ffbc
- . dccc
- 3. dbcc
- 3 dbbc
- 4. bbbc
- 6. bbcc
- 4 bccc
- . cccc

36.

— aaaa — daaa + ffaa — ggga == bbbb.

A:
- — ggg
- ff
- — d
- — bbb

Ab:
- — gggb
- . ffbb
- — dbbb
- — bbbb

B:
- — . ggg
- . ff
- . 2 ffb
- — . . d
- — 3. db
- — 3. dbb
- — 4. bbb
- — 6 bb
- — 4 b

Bc:
- . . gggc
- . . ffcc
- . 2. ffbc
- — . . dccc
- — 3 dbcc
- — 3. dbbc
- — 4. bbbc
- — 6. bbcc
- — 4. bccc
- — . cccc

37.

aaaa — daaa + ffaa + ggga ═══ bbbb.

A { · ggg
· ff
— d
· bbb }

Ab { · gggb
· ffbb
— dbbb
· bbbb }

B { · · ggg
· · ff
· 2. ffb
— · d
— 3. db
— 3 dbb
· 4. bbb
· 6. bb
· 4. b }

Bc { · · gggc
· · ffcc
· 2 ffbc
— · dccc
— 3. dbcc
— 3. dbbc
· 4 bbbc
· 6 bbcc
· 4. bccc
· · cccc }

38.

— aaaa + daaa — ffaa — ggga ═══ bbbb.

A { — ggg
— ff
· d
— bbb }

Ab { — gggb
— ffbb
· dbbb
— bbbb }

B. { — · · ggg
— · ff
— 2. ffb
· · · d
· 3. db
· 3. dbb
— 4. bbb
— 6. bb
— 4. b }

Bc { — · · gggc
— · ffcc
— 2 ffbc
· · · dccc
· 3. dbcc
· 3. dbbc
— 4 bbbc
— 6 bbcc
— 4. bccc
— · cccc }

39.

aaaa + daaa — ffaa — ggga ═══ bbbb.

A { — ggg
— ff
· d
· bbb }

Ab { — gggb
— ffbb
· dbbb
· bbbb }

B { — · ggg
— · · ff
— 2. ffb
· · · d
· 3. db
· 3. dbb
· · 4 bbb
· 6. bb
· 4. b }

Bc { — · gggc
— · · ffcc
— 2 ffbc
· · dccc
· 3. dbcc
· 3 dbbc
· 4 bbbc
· · 6. bbcc
· 4. bccc
· · cccc }

40.

— aaaa — daaa + ffaa + ggga ═══ bbbb.

A { · ggg
· ff
— d
— bbb }

Ab { · gggb
· ffbb
— dbbb
— bbbb }

B { · · ggg
· · ff
· 2. ffb
— · · d
— 3. db
— 3. dbb
— 4. bbb
— 6. bb
— 4. b }

Bc { · · gggc
· · · ffcc
· 2. ffbc
— · · dccc
— 3. dbcc
— 3. dbbc
— 4. bbbc
— 6. bbcc
— 4 bccc
— · · cccc }

41.

aaaa − daaa + ffaa − ggga ═ bbbb.

A { —ggg / .ff / —d / .bbb }

Ab { —gggb / .ffbb / —dbbb / .bbbb }

B { —.ggg / ..ff / .2ffb / —.d / —3.db / —3.dbb / .4.bbb / .6.bb / .4.b }

Bc { —..gggc / ...ffcc / .2.ffbc / —..dccc / —.3.dbcc / —3.dbbc / .4 bbbc / .6 bbcc / .4.bccc / ..cccc }

42.

− aaaa + daaa − ffaa + ggga ═ bbbb.

A { .ggg / —ff / .d / —bbb }

Ab { .gggb / —ffbb / .dbbb / —bbbb }

B { ..ggg / —.ff / —2ffb / ..d / .3.db / .3.dbb / —4.bbb / 6.bb / 4.b }

Bc { ..gggc / ..ffcc / —2ffbc / ..dccc / 3.dbcc / 3 dbbc / —4.bbbc / —6.bbcc / —4.bccc / —.cccc }

43.

aaaa — daaa — ffaa + ggga ═ bbbb.

A { .ggg / —ff / —d / .bbb }

Ab { .gggb / —ffbb / —dbbb / .bbbb }

B { ..gggg / —.ff / —2ffb / —.d / —3.db / —3.dbb / ..4 bbb / 6.bb / 4.b }

B { ..gggc / —.ffcc / —2ffb / —.dccc / —3.dbcc / —3 dbbc / .4 bbbc / 6.bbcc / .4 bccc / ...ccc }

44.

− aaaa + daaa + ffaa − ggga ═ bbbb.

A { —ggg / .ff / .d / —bbb }

Ab { —gggb / .ffbb / dbbb / —bbbb }

B { —.ggg / ..ff / ..2.ffb / ....d / .3.db / .3.dbb / —4.bbb / —6.bb / —4.b }

Bc { —.gggc / ..ffcc / .2.ffbc / ..dccc / .3.dbcc / .3.dbbc / 4.bbbc / —6.bbcc / —4.bccc / —.cccc }

45.

$aaaa - daaa - ffaa - ggga = bbbb.$

A
— ggg
— ff
— d
. bbb

Ab
— gggb
— ffbb
— dbbb
. . bbbb

B.
— . ggg
— . ff
— 2. ffb
— . . d
— 3. db
— 3. dbb
. 4. bbb.
. 6. bb
. 4 b

Bc
— . gggc
— . ffcc
— 2. ffbc
— . dccc
— 3. dbcc
— 3. dbbc
. 4 bbbc
. 6. bbcc
. 4 bccc
. . cccc

46.

$-aaaa + daaa + ffaa + ggga = bbbb.$

A
. ggg
. ff
. d
— bbb

Ab
. gggb
. ffbb
. dbbb
— bbbb

B
. . ggg
. . ff
. 2 ffb
. . d
. 3. db
. 3 dbb
— 4 bbb
— 6. bb
— 4 b

Bc
. . gggc
. . ffcc
. 2. ffbc
. . dccc
. 3. dbcc
. 3. dbbc
— 4. bbbc
— 6 bbcc
— 4. bccc
— . . cccc

Sic igitur exacta est æquationum de triplici genere proposito cum canonibus suis directorijs vniuersa descriptio: Quæ, vt Problematum superiorum Corollarium generale, & ad Exegeticen numerosam maximè conueniens Appendix, secundæ & exegeticæ tractatus huius parti finem imponat.

Ad Mathematices studiosos.

EX omnibus *Thomæ Harrioti* scriptis Mathematicis, quòd opus hoc Analyticum primum in publicum emissum sit, haud inconsultò factum est. Nam, quùm reliqua eius opera, multiplici inuentorum nouitate excellentia, eodem omninò quo tractatus iste (Logistices speciosæ exemplis omnimodis totus compositus) stilo Logistico, hactenùs inusitato, conscripta sint, eâ certè ratione fit, vt prodromus hic tractatus, vltra proprium ipsius inæstimabilem vsum, reliquis *Harrioti* scriptis, de quorum editione iam seriò cogitatur, pro necessario præparamento siue introductorio opportunè inseruire possit. De quâ quidem accessoriâ operis huius vtilitate rerum Mathematicarum studiosos paucis his præmonuisse operæprecium esse duximus.

	Errata.	*Sic corrigenda.*
In Præfatione		
Pag 1. lin. 17.	illum certamine	illum in certamine
In definitionibus		
Def. 6.	tantum	tanquam.
Def. 8.	dato scil.	datum scil.
Def. 15.	$+ faaa$	$- faaa.$
Def. 17.	in quartâ	in quintâ.
In Sectionibus.		
Pag. 18. Prop 5. quinquies repetitum	$-bca$ $-bda$ $-cda$	$+bca.$ $+bda.$ $+cda.$
Pag. 27 lin. 6.	& 10.	& 18.
Pag 29 lin. 6.	ba	$aa.$
Pag. 32 lin. 13.	$+bca$	$aaa + bca.$
Pag. 32 lin. 35.	trinomia	binomia.
Pag 36. lin. 5.	$+ bfaa$	$- bfaa.$
Pag 53. lin. 20. bis	$-d$	$+d$
Pag. 72. lin. 20.	per 22.	per 21.
Pag. 75 lin. 22.	b. vel dici	b. vel c. radici
Pag. 77 lin. 19	a. in erit	a. in c. erit
Pag. 81. lin. 5. 6.	$+ qqr\vert$ $\frac{+ qrr\vert}{4.}$	$qqr + qrr\vert$ $\frac{qqr + qrr\vert}{4.}$
Pag. 81. lin. 22.	$qq + qr + rr$	delenda.
Pag 84. lin. 20.	$pq + pr + qr$	$\frac{pq + pr + qr}{3.}$
Pag 99 lin. 4.	$\sqrt{}.ccc + \sqrt{}.-cccccc$	$\sqrt{}.-ccc + \sqrt{}.cccccc$
Pag. 99. lin 6.	$\sqrt{}.ccc + \sqrt{}.-cccccc$	$\sqrt{}.-ccc + \sqrt{}.cccccc$
Pag. 99. lin. 7.	$-\sqrt{}.ccc + \sqrt{}.cccccc$	$\sqrt{}.-ccc + \sqrt{}.cccccc$
In Exegetice numerosâ.		
Pag 117. lin. 7.	reducere	educere.
Pag 126 lin. 8.	Ab . . . 1518.	$-Ab$ . . . $-1518.$
Pag. 126. lin. 17.	Bc . . . 118.	$-Bc$ . . . $-118.$
Pag. 128. lin. 16.	$b = 5.$	$c = 5.$
Pag. 128 lin. 22.	$aa + da$	$-aa + da$
Pag. 129. lin. 30.	1173	$-1173.$
Pag. 130. lin. 11.	939.	$-939.$
Pag 140 lin. 13.	2 7 4 5 7 6	2 7 4 5 $\bar{7}$ 6
Pag 140 lin. 14.	. . 7 5 0 0	7 5 0 0
Pag 140 lin. 15.	. . 1 5 0	1 5 0
Pag. 148. lin. 19.	$\underline{-3.bcc}$ . . . $\underline{-960}$	$-3.bcc$ . . $-960.$
Pag. 148. lin. 20.	$-ccc$ . . . $-64.$	$-ccc$ . . $-64.$
Pag. 155. lin. 2.	$\underset{\bullet}{7}$ 8 $\underset{\bullet}{8}$ 0 $\underset{\bullet}{8}$	7 8 $\underset{\bullet}{8}$ $\underset{\bullet}{0}$ $\underset{\bullet}{8}$
Pag. 164. lin. .	$b = 6$	$c = 6$
Pag. 164. lin. 35.	quaternarijs in quadratico	quaternarijs in biquadratico.